Becoming bi-numerate: a study of teachers' practices concerning children's early 'written' mathematics

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Abstract:
Rather than being a transparent symbol system, young children first meet written mathematics as an unfamiliar foreign language, acknowledged as causing considerable difficulties for their understanding of mathematics (Ginsberg, 1977; Hughes, 1986; Williams, 199, Worthington & Carruthers, 2003).

This study focuses on responses from three hundred teachers of children 3 – 8 years through questionnaires and telephone interviews. Our findings show that whilst research and current guidance emphasises the value of encouraging children to use their own marks, teachers find this highly challenging and a significant percentage rely instead on published worksheets and 'closed' ways of recording.

In this paper we shall argue that children's informal 'written' mathematics is a strength which they can use. They become what we have termed 'bi-numerate', enabling them to co-construct their understanding of the role and function of symbols at a deep level, through socio-cultural contexts. Their visible marks provide rich opportunities for children and teachers and are the key to later success in mathematics.

Keywords: Mathematical graphics; abstract mathematical symbols; worksheets, meaning; Bi-numeracy

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Understanding symbols

Our interest in children's mathematical mark-making grew from our classroom teaching and study of young children's early writing development, part of our practice for many years. Such an approach is also termed 'emergent' or 'process writing'. The many studies of this area of learning suggest ways in which teachers support the growth of understanding for example, Clay, 1975; Holdaway, 1979; Smith, 1982; Hall, 1987; Cambourne, 1988; Barratt-Pugh and Rohl, 1997. Like literacy, the foundations of numeracy are embedded in young children's early experiences as they 'undergo unparalleled cognitive, social and emotional growth' (Diezman & Yelland, 2000, p. 48). However, there has been a dearth of studies that have focused on children's informal marks and personal written methods and their personal meaning-making in mathematics.

It has been well documented that the key to young children's understanding of formal mathematics, is to support them so that they form links between their own informal mathematics and the abstract symbolism of school-based mathematics. As early as 1977 Ginsberg argued that the gulf between children's invented strategies and school-taught, formal written procedures was a very likely cause of children's difficulties with school mathematics, a view echoed by Allardice (1977). Heibert (1984) had a similar thrust, arguing that making connections between formal school mathematics and the children's own mathematics was imperative, if children were to become mathematical thinkers rather than mindless followers of mechanistic rituals. In 1986, Hughes's research highlighted the gap between home and school mathematics, showing how young children could use informal marks in meaningful ways. Hughes's research appears to have influenced the writers of official curriculum documents in England (e.g., QCA, 1999 and 2000). However, as our study shows, Hughes's influence on teaching has been sparse.

Since Hughes's study in 1986, there have been several individual accounts of children using their own mathematical marks: in the USA, Whitin, Mills and O'Keefe (1990); in Australia, Stoessinger and Edmunds (1992) and in England teachers' stories of their work (Atkinson, 1992). This was the beginning of a movement in the direction of advocating what we term as a 'bi-numerate' approach to the teaching of mathematics. Whitebread argues that 'what is clear is that children cannot be encouraged to use new strategies very effectively simply by being taught them as an abstract procedure (1995, p.35). Gifford also supports children's early informal mathematical representations, contending that 'it allows children to make sense of ideas by representing them in their own way' (Gifford, 1997, p.86). More recently reports in Australia (Bobis et al 1999 & Doig et al, 2003) support this thesis, arguing that teachers need to 'build on children's current knowledge base' since children's informal and intuitive numerical ideas… 'form a very important basis for…(future) development' (Bobis et al, 1999, p.134). Hunting proposes that 'meaningful mathematics learning occurs when each child associates some personal experience… negotiated through social experience with others – with symbols' (1999, p.80). Successive government reports in England clearly support this thesis (e.g. DfES, 2000 & 2002; HMI, 2002) although there has been little focus on this aspect of young children's meaning making and no evidence that official guidance for the youngest children has transferred to the classroom.

Teachers' difficulties

We argue that three key difficulties have restricted teachers in their understanding:

- whilst official guidance (QCA, 1999b & 2000) supports children's own marks and written methods in specific statements, guidance on how teachers might support this has been lacking
- the number of published findings is limited. Past research failed to make links between research and practice
- significantly, research findings often take many years to filter through to classrooms, or fail to do so at all (Hopkins, 2002)

For teachers it is easy to disregard young children's early mathematical marks as incomprehensible or poor (Litherand, 1997). Even when children explain their thinking, teachers can still be presented with a dilemmas because it may look wrong or appear untidy; they may have crossed things out or chosen not to use standard procedures. Teachers may feel that it takes too long to ask the child about it since it takes time to tune into children's thinking. In classrooms where children are not given the opportunity to put their thoughts on paper, it never happens anyway. In such classes children's mathematical graphics do not fit the norm, and for many pressurised teachers it is too much to cope with: it disturbs the equilibrium. And teachers are often too busy to reflect on what the marks might mean. Yet a decision to explore further the meaning of their marks could render different perceptions. As Litherand emphasises:

> …for the teacher who views learning as a process of development and construction rather than a process of association, knowledge could be seen as of personal and social construction rather than as fixed and immutable, as dynamic rather than static. The impact of such differences upon the criteria by which teachers judge achievement is significant (1997, p.11).

Supporting children's early writing development is problematic for some teachers and it appears that introducing abstract symbolism of mathematics is more so. As Heibert observes, 'even though teachers illustrate the symbols and operations with pictures and objects, many children still have trouble establishing important links' (1984, p.501). Determining ways to foster these connections has been a challenge for teachers but as Hughes (1986) and others have observed, a failure to do this is likely to be where many children's difficulties lie.

Whilst recent research into children's own written methods has focussed on older children's written methods, (e.g. Anghileri, 2000 and Thompson, 1997), extensive evidence-based research in real homes, nursery classes and schools had not been carried out, nor has the development of young children's own mathematical marks and written methods been fully researched until now (Worthington & Carruthers, 2003a). Thus there has been a lack of 'joined-up' thinking. Although official guidance is unequivocal, without practical advice for teachers in England on ways in which children's informal mathematical marks might be supported, this aspect of mathematics teaching will remain largely unexplored for children of 3 – 8 years.

Mathematical graphics

'Mathematical graphics' is the term we originated to describe the full range of marks children make when exploring their mathematical thinking. Graphical marks concern visual communication through 'making or choosing marks and arranging them on a surface to covey and idea… signs whose context gives them a unique meaning, and whose positioning can lend them a new significance' (Hollis, 1996, p.7). Mathematical graphics fall within the context of what is known as 'mark-making,' a term used to describe marks that include both early writing and drawing and is now rightly regarded as just one form of representing meaning which may be made in multi-modal ways (Kress, 1997). Making marks on paper or with other resources, children are constructing meaning through their representations: this allows them to make 'child sense' (Carruthers, 1997, p.13) of the abstract symbols.

Representing meaning

From a Vygotskian perspective, symbols or graphical representations bridge the gap between 'enactive, perception-bound thinking and abstract, symbolic thinking' (van Oers, 1997, p. 237). De Loache (1991) observed that children as young as 18 months can pretend that 'a block of wood is a car, or that a banana is a telephone': in doing this they are able to represent something in two different ways. This flexibility of meaning and object allows children to understand that marks - or written symbols on a page - stand for something other than what they appear. For example, in our culture a cross drawn on a card or paper may first become associated with a kiss but share a multiplicity of meanings in other contexts.

Deloache et al point out that 'no symbol system is fully transparent' Letters of the alphabet and numerals, for example, 'have no inherent content or meaning, but convey information in systematic ways'(1998, p. 325). Young children not only have to make sense of individual symbols but need to understand their role within a system whether for example, letters within a written word, marks that denote parts of a drawing or a mathematical symbol within a written calculation. Understanding abstract symbols in written language or mathematics begins long before children enter school: they have a 'pre-history' that Vygotsky believed originates in both gesture and the alternative meanings that children assign to objects within their play.

There is considerable pressure placed on many young children to achieve drawing, writing and 'written' mathematics that is 'correct' and of a standard form - before they are able (see for example, Matthews, 1999; Anning, 2002). This has resulted in extensive copying, cutting and sticking, colouring-in and filling boxes. Such 'products' allow scarce opportunities for children to make meaning and result from a failure on the part of many to take account of young children's perspectives. Dockell & Teubell argue that whilst 'young children distinguish different domains of symbol knowledge knowing (writing, numbers and drawing) before they can accurately produce domain-relevant symbols', this distinction is not apparent in their `use of notation as a communicative tool' (in Munn, 2001, p.35).

It is significant that in children's mathematical graphics, children decide for themselves how best to represent their thinking. And, as children mature, their graphics increasingly show evidence of complex levels of thinking. We argue that young children's early representations must be recognised as the considerable achievement that they are. To borrow Bakhtin's definition, the child 'populates it with his own intention, his own accent, appropriating and adapting it to his own semantic and expressive intention' (Bakhtin, 1981, pp. 293 – 294). For children of three to eight years, personal ways of representing mathematics are powerful indeed, providing windows on their minds that can illuminate our understanding if we only watch, listen and value what they do.

Understanding mathematical symbols

The ways in which we set down mathematical symbols can cause confusion for young children, for example the numerals 6 and 9 may appear the same since one is the inverse of the other in appearance. Place value causes problems, for example the numeral '2' has a different value form the two in '25' and some signs look similar, such as the equals and subtraction signs. To further complicate matters, children are also learning about the two symbol systems of writing and mathematics at the same time (DeLoache, 1991). It could also be argued that the writing system makes more sense to children. When given the choice some children prefer to use writing instead of mathematical symbols (Pengelly, 1986). It is no wonder that young children find it difficult to navigate the various symbol systems. Moreover, whilst spoken language can be ambiguous – dependent whether it is used in a natural context or a specifically mathematical one, it appears that written (graphical) symbols may be even more so. Twenty-first century culture draws heavily on the visual impact of advertising, logos, photographs, film, cartoons, packaging and computer-generated images and there is every indication that this is a significant feature of children's culture.

Ginsberg (1977) lists three principles of children's written symbolism in mathematics:

- understanding of written symbolism generally lags behind children's informal arithmetic
- children interpret written symbolism in terms of what they already know
- good teaching attempts to foster connections between the child's informal knowledge and the abstract and arbitrary system of symbolism

Whilst it is clear that young children do have considerable informal knowledge of mathematics by the time they enter nursery or school as our research has shown (Worthington & Carruthers, 2003), Ginsburg's last point is where the difficulties lie. There has been what Munn describes as 'a considerable gap in our knowledge of just how young children develop the ability to use number symbols… and the development of children's use and understanding of written numerals' (Munn, 2001, p. 35).

In our research we have not only explored children's development and use of written numerals and mathematical signs, but shown how children also experience difficulties with standard algorithms during their first years in school. We show how supporting children's informal written calculations, allows them to construct understanding of the processes and integrate personal meanings about symbols. It also enables them to make valuable links between their own mental and written methods. This in turn this leads to deepened understanding of subsequent standard algorithms and an ability to make effective decisions when tackling calculations and solving problems.

Teachers' practice concerning children's early 'written' mathematics

This study was only one part of our research into children's mathematical graphics. We wanted to ascertain the scope for children to use their own mathematical marks in Early Years settings. Such opportunities are governed by decisions made by teachers – whether to use mathematics worksheets or to provide blank paper. Determining teachers' practice in 'written' mathematics would also highlight the extent to which recommendations of the National Numeracy Strategy and the Foundation Stage had influenced their practice. We collected the data during 2003, four years after the launch of the National Numeracy Strategy. This aspect of our research was in two parts:

1. Questionnaires
Our responses were gathered during a one-year period, from 273 teachers in four areas of England. Three areas were large cities: in the north, west and south-west of England. The remaining area was a largely rural county in the south of the country.

We chose to use questionnaires since they can 'provide data economically and in a form that lends itself perfectly to the purposes of (our) study' (Verma & Mallick, 1999. p.117). We hoped the findings would provide us with information about adults' expectations and the opportunities that children had to represent their mathematical thinking in 'personally meaningful ways' (van Oers, 2003). However we made a mistaken assumption, expecting that using blank paper would provide children with open opportunities that we believed would help them make their own meanings. When we analysed the responses we were very surprised by the results.

Our initial questionnaire was piloted with a group of teachers and subsequently adjusted by reducing the number of questions to allow us to analyse a larger sample of responses. Answers to closed questions were coded, allowing straightforward quantitative analysis regarding use of worksheets and blank paper. Our questionnaire focused on two key aspects: We asked:

- Do you give children worksheets for mathematics?
- Do you give children blank paper for mathematics?

We also invited teachers to give examples of the sort of things the children might do, either on worksheets or blank paper: these were more open questions, providing a range of qualitative data which from transcriptions. We were then able to analyse this rich source of data in some detail.

We distributed questionnaires at either Early Years mathematics conferences or seminars, or continuing professional development courses (CPD) on mathematics teaching at which we were speaking, at the beginning of the day. This was to ensure that responses were not influenced by anything we subsequently said in the speech or seminar we gave. We allowed time for teachers to complete questionnaires and emphasised that their involvement was both voluntary and anonymous. Ninety-six per cent of teachers present on these days completed questionnaires, a high return related to the fact that teachers were given time to complete questionnaires on the day and did not need to remember to post them.

2. Telephone interviews

Because there are limitations in what can be achieved through the use of questionnaires (Brown & Dowling, 1998, p. 66), we decided to conduct interviews as a follow-up to the questionnaires.

We had invited teachers willing to be interviewed by phone, to add their first name and a home phone number to their questionnaire: seven per cent of those who had completed questionnaires did this. We used individual's questionnaires as a starting point for interviews and these were an approximately forty-five minutes long and semi-structured, using a combination of closed and open questions. Interviews were conducted a term after the conferences and courses, allowing us to explore the extent to which teachers had developed their thinking and practice. They allowed teachers to 'use their own words and develop their own thoughts' which as a research method is a 'good way of discovering thinking about complex issues... (allowing) in-depth exploration of personal experiences and feeling' (Descombe, 2003. p.167). As researchers we shared responsibility for conducting interviews and discussed the outcome of each interview immediately afterwards. Combining this with analysis of transcribed responses supported the validity.

It is important to point out that all teachers who completed questionnaires had elected to attend the conferences or courses. Our sample was therefore most likely to include teachers who appeared to have a general interest in supporting young children's mark-making and early writing, and in mathematics specifically. Those who volunteered to be interviewed on the telephone may be considered to have even higher levels of commitment to the teaching of mathematics in the Early Years. Certain aspects of the findings are therefore even more surprising in the light of this interest.

Outcome of the study

1. Worksheet use

It was evident that there was a large difference in use of worksheets when comparing different types of Early Years settings and classes with different ages of children (see table 1.1, below).

Table 1.1. Results of teacher questionnaires

Type of setting	*Percentage using worksheets*
Maintained nursery classes	20
Private nurseries	63
Pre-schools (voluntary)	72
School classes with 4/5 year olds	89
School classes with 4 – 6 year olds	100
School classes with 7 year olds	100
School classes with 8 year olds	100

Children in maintained (state run) nursery classes appeared to be freer in terms of their 'written' mathematics, whilst staff in voluntary run pre-schools made the greatest use of these published materials for children less than 5 years of age. Once children arrived in school – in England generally at the very early age of 4 years old – almost 90 per cent of the teachers in our study use worksheets and by the Y1 all teachers in our study used them. Four year olds who were in mixed aged classes with 5 – 6 year olds were more likely to use worksheets than children in classes of only 4 and 5 year olds.

Our findings regarding use of worksheets are consistent with what Millett and Johnson observe is a 'world-wide trend'; 'typical teaching is assumed to be the total or significant use of a commercial mathematics scheme' (cited in Maclellan, 2001, p. 76). The recommendations for teachers regarding children's early written methods in the Foundation Stage (QCA, 2000) and for primary teachers (QCA, 1999) are not reflected in our research findings. Children's own mathematical mark-making is supported in the guidance, for example, 'children's own recordings… form an important stage in developing fluency'(QCA, 1999. p.12) and 'children will need to have plenty of experience of using their own ways of recording' (QCA, 1999. p. 19). Currently teachers are unsure how they might put recommendations about children's written methods into practice – particularly for children between the ages of 3 and 7 years. The concern about this extensive use of worksheets is growing and is one we discuss below.

Worksheet content

- Common examples of content cited by teachers in our study included (for both Nursery & Reception): 'copying numbers'; 'cutting and sticking'; 'dot-to-dot'; 'colours'; 'pencil control'; 'filling in missing numbers'; 'colouring in shapes'; 'colouring the bubbles on the snake the correct cuisenaire colour' and draw six apples on the tree'.
- Examples for Y1 included those already cited for the younger children and 'placing numbers on the caterpillar in the right order – counting forwards and back'; ' use 'clever sticks' to make a 'Funnybones' animal – record carefully using correct colours and count'; 'filling in missing numbers on the bean stalk and 'pre-written sums, '+' and '-' and '□ + 5 → 6; 1 + □ →, etc.,'. Other Y1 examples were of estimation (toy dinosaurs in a cave); giving totals for the number of coins in a purse and sorting straight and curved sided shapes.
- In Y2, some teachers were still providing worksheets with 'matching activities' and 'drawing shapes' although most featured 'four rules work' such as 'take-aways'.

These examples can be divided into three types:

1. Low level activities of the 'fine-motor' skills type that have little or no mathematical content
2. Some worksheets for children in Reception and Y1. appear to have been chosen for what adults view as appealing to children, such as pictures of caterpillars, 'Funnybones' and a beanstalk.

(Neither of these types of worksheets (see points 1 and 2) are likely to challenge children's thinking and engage them mathematically at a deep level.)

3. Worksheets with adult-designed algorithms using standard symbols. However, since children are unable to make any decisions about how they might represent their thinking, they are prevented from making any connection between their mental and 'written' methods of calculations. Most significantly, they prevent children creating their own meanings and because each calculation only has one 'right' way of working out and representing: this negates the co-construction of meanings within socio-cultural contexts.

A few teachers felt uncomfortable about admitting that they used worksheets, for example, 'we use worksheets though we are trying to cut down' whilst another enthusiastically noted 'all our worksheets are different!' Confusion over the role of representation in mathematics clearly persists, exemplified by comments such as 'we are not allowed to do worksheets in the nursery – practical activities instead'.

2. Using blank paper

Whilst 79 per cent of teachers of 3 – 8 year old children in our study used worksheets, 82 per cent of the same teachers also either allowed or encouraged some use of blank paper for mathematics. At first glance these figures appear to be encouraging. Teachers who said they used blank paper provided almost 500 examples of the sort of mathematical marks children might make (an average of two examples per teacher). However, of these, most (almost 85 per cent) were either when the teacher told the children what to do and how to record. Some teacher had hand written on blank paper, what was in effect, a copy of a worksheet. This 'copied' worksheet, teachers said, was very time-consuming but suited their purpose more than commercially produced worksheets. Using blank paper in this way has the same pitfalls as worksheets but the teachers believed them to be more personal to the class and for supporting curriculum objectives. This kind of recording lies in the same category as teacher-made worksheets. We would argue that this fails to give a true account of where the children are in their written mathematics and how much they understand.

The ‘content’ of the mathematics that children explored on blank paper was no more challenging than that on worksheets, with the same emphasis on following instructions and completion, rather than on thinking and making mathematical meaning.

Teachers’ examples of mathematics on blank paper

- In Nursery and Reception children were asked to repeat patterns that a teacher had begun; draw round shapes; do ‘cutting’; ‘matching’; ‘folding’; ‘number rubbing’ and copy numbers; ‘have a go at writing the number of the week’ and ‘draw pictures of a ‘ lego’ model – how many did you use?’.
- By Y1. teachers were asking children to record a practical activity such as a dice game ‘in the form of number statements’ and ‘practice writing number sentences’. During Y1. some of teachers’ use of blank paper showed increased emphasis on calculations, ‘using apparatus for addition’ and ‘record different ways of making 10p, 15p etc.’, although activities such as ‘coin rubbings’; ‘draw a model and record showing numbers, colours and shapes’ persisted.
- The Y1. drawing activity was still evident in some Y2. classes, as was ‘making a repeating pattern’ and ‘making a picture using shapes’.

It is also significant that neither the content of these 500 examples of worksheets nor what the children did on blank paper appear to support the guidance set out by QCA. in terms of teaching written calculations.

3. Mathematical mark-making within play

Nine per cent of examples referred to blank paper within children’s role play, providing positive opportunities for children to use marks within the context of their play. Unfortunately in the current context in England role-play is rarely found in classrooms with children beyond the age of five, effectively ruling out the use of blank paper within any play situations for children once they move into their second year in school.

In only 6 per cent of examples of mathematics on blank paper did teachers refer to children ‘making their own marks’; using ‘emergent writing’; using paper ‘in their own way’ or making ‘jottings’: jottings are short written forms to aid mental methods such as the use of an empty number line, introduced through the National Numeracy Strategy in England. This paints a disappointing picture, especially since most of these teachers used worksheets for a greater part of the time. Blank paper appeared therefore to be viewed as an extra to the teachers’ usual activities or for occasional use.

4. The status of children’s informal marks

We wondered what happened to the pieces of paper on which the children made their own marks. In our own teaching we have found that these pieces provide valuable information that helps us assess, support and extend children’s learning and build a constantly unfolding picture of their development. When asked, 77 per cent of those interviewed said that the children took home what they had done and several replied that they did not keep them. Of those interviewed who did occasionally use blank paper, only 23 per cent kept examples with children’s marks. These were stored as a record but not used for assessment purposes, and only a very small sample of what the children had done was saved. Blank paper was therefore more likely to be used only occasionally and its use was of a significantly lower status to worksheets. The marks the children made on paper during their play, for example, were generally not saved by teachers: in some instances teachers commented ‘I bin them’ (i.e throw them in the waste paper bin). This shows that these were not seen as significant in contributing to the children’s developing understanding. We also appreciate the huge pressures that teachers are currently experiencing which limit reflection. It appears that a widely held view is that children’s mathematics on paper is significant only when it is the outcome of a teacher-directed activity.

These findings point to written mathematics that is largely on worksheets or following the direction of the teacher. Furthermore, we believe that it must be almost impossible to trace children’s development unless a comprehensive, dated collection of pieces is kept. In our work in visiting many Early Years settings we also seldom see examples of children’s own mathematical marks displayed - something that could provide readily available peer models of a range of ways of making meaning and emphasise the value of diverse ways of thinking and representing mathematical meaning.

The problem with worksheets

Selinger argues that schemes have controlling material that determines what should happen next and what pathways of learning should be encouraged (Selinger, 1994). They generalise for all children and provide a dependent culture for the teacher as well as the child. Anghileri is concerned that schemes often introduce set procedures and formats (Anghieri, 2000). The children see calculations as ‘rituals’ which leads to little understanding of the symbols used.

Pound discusses the prolific use of worksheets in Early Years settings, arguing that ‘worksheets are seen by many Early Years workers and parents as being an indication of a formal and somehow more productive educational process going on’ (Pound, 1998, p.13). She argues that suppliers suggest that among other things, the sheets may be used to introduce the child to recording. Pound strongly refutes this idea by arguing that the restricted format of the worksheet does not encourage the children’s own meaning but sets a ‘tight jacket’ which hampers their drawing and writing (Pound, 1998). Fisher agrees: ‘worksheets restrict what a young child can tell you about what they know and understand. If children devise their own ways of recording knowledge and understanding, then they will select ways which make sense to them and give all the information they want to share’ (Fisher, 1996, p. 59).

But it may also be that the breadth of young children’s experience of mathematics in the Early Years fails to challenge, whether practical or early written mathematics. A recent study of children's experiences in Reception classes (Adams et al. 2004) highlighted a need for teachers to 're-examine the values and priorities of their approaches to numeracy teaching (p. 27) calling into question the limited value of certain aspects of teaching mathematics. The authors of this report emphasise ' the low level of cognitive demand in that the children were being asked to do little more than recall, label and repeat’ in the teaching of numeracy in Reception (p. 85). A major new study of mathematics in the Early Years (EPPI, 2004) raises questions concerning the way in which our youngest children experience mathematics and the way in which it is taught. It points to an emphasis on 'teaching to the test' and teachers' difficulties that result in lessons that 'fail to challenge and extend the thinking of their pupils' (Slater, 2004, p.6). Such limitations on children’s thinking are clearly an issue. This forthcoming report calls also for in-service training for teachers: this may be just what is needed - unless all that is offered is more of the same.

To provide opportunities for high-level cognitive demand children need to explore their thinking through their own representations rather than merely recording the outcome of a practical activity, colouring or tracing. We have tried to show something of the level of challenge and thinking that children experience when working in more open ways and when selecting their own written methods (Worthington & Carruthers, 2003a; Carruthers & Worthington, 2005). We are concerned with mathematics that goes beyond 'performance' and 'products': children deserve more than basic skills teaching that lack genuine challenge and opportunity for creative thinking.

As Cullen and St George point out that, 'when teachers over-emphasise teacher directed tasks such as worksheets, children view learning as dependent on the teacher' and are 'a recipe for failure' (Cullen and St George, 1996, p. 4). For children, being dependent on the teacher is directly related to 'helplessness' behaviours where children's develop negative views of themselves as learners and consequently, their ability to meet challenges and to succeed (Dweck & Leggett in Sylva, 1993).

There is also concern that for children with learning difficulties that their 'mathematical experiences (are) centred around worksheets' (Robbins, 2002, p. 133). For many children with special needs, worksheets are a recipe for failure. However, when children record in their own way, because they have to think carefully about what they do and consider a range of possibilities, they will learn more about the mathematics. It is because of this struggle that occurs within their minds that 'they will do better than their perceived best' (Brighouse and Woods, 1999, cited in Robbins, 2002, p.5). Furthermore, Newman argues that 'activities that involve fragments of language, that discourage children taking chances, that don't permit the exchanging of idea, can only serve to make reading and writing more difficult' (Newman, 1984, p. 72).

Brooker's powerful study of children learning the school's culture when they first enter the reception class highlights some of the problems:

> Systematic observations… suggest also a particularly low level of commitment while children are occupied with worksheets. Most children treat the completion of such sheets as a self-standing task unconnected to any to any learning activity with which they may be associated in a teacher's mind. Rather, the sequence of actions involved in 'doing a worksheet' appears to be one of the more mysterious aspects of learning to be a pupil….
>
> The regular use of worksheets suggests a partial, reluctant surrender to the modality of the performance model, with its emphasis on product, economy and accountability. The activity is at once strongly framed as a regulative discourse (children required to sit at a table and complete the tasks when summoned) and weakly framed as an instructional discourse (children have little idea what knowledge or skills they are acquiring, or demonstrating). An observer might conclude that worksheets represent the worst of both worlds: they incur the disadvantages of the performance model without realizing any of its possible advantages for children's learning (Brooker, 2002. p.87).

The need to work from the child can help us to know where to start supporting children's own mathematics and to respond and build on children's 'home and community literacies' (Makin, 2003. p. 97) although studies such as Aubrey's (1994) and Munn's (1994) show that children's informal mathematical understanding is not always acknowledged on school entry. Munn observes that researchers elsewhere have arrived 'at the same conclusion about the relationship between children's pre-existing knowledge and the school curriculum', including Irwin (1996) and Young-Loveridge (1989) in New Zealand and Wright, (1994) in Australia.

Significantly, both published and teacher-made worksheets prevent children in making sense of the standard, abstract symbolism of mathematics since they deny opportunities to make meaning through their own marks and written methods and to 'translate' between their informal symbols and the standard forms, which we term 'bi-numeracy'. Worksheets also perpetuate a 'right/wrong' culture based on behaviourist theories, a view of mathematics opening mathematics to a fallibilistic perspective (Ernest, 1991). Cockburn believes that when children enter school they often learn to 'play the (mathematics) game' where the emphasis is on finding the right answer – which is always the teacher's answer (1999, p. 9). It is also significant to note that in fourteen years of research, we have not found one research paper that provides evidence of support for the use of worksheets.

Children's own ways make sense to them. In discussing children's own mathematics with them we are often excited and surprised by the extent of their understanding. And for adults, children's own mathematics provides a 'discussion paper' – a shared resource for negotiating meaning.

Assessing mathematics

The issue of teachers' responses in our study that they used worksheets 'for assessment purposes' is particularly puzzling: as Brooker observes, 'most worksheets exist simply to record a child's performance on a particular day, providing low-level evidence of quite low-level skills such as tracing and colouring' (2002, p.87). We argue that worksheets are probably the *least* likely to provide evidence of children's understanding since:

- the children have no ownership of the content of the worksheet
- they are confined by layout; the child has to fit into the worksheet organisation and way of doing mathematics
- most worksheets have closed questions and only one answer; this may make the situation a testing one for young children
- worksheets do not tell what a child knows about mathematics and the way they are thinking
- worksheets do not reveal what the child can do but often what they cannot do
- young children can become bewildered in finding the sense in a worksheet
- the match of worksheet to child is difficult and children can work below the level of their true ability
- worksheets that claim to be 'teaching' mathematics have sometimes very little mathematics in them to assess; for example, a typical worksheets may have the numeral two in dots to trace over, accompanied by two large balloons to colour in. The child might respond to the teacher's question, 'how many balloons?' This takes a few seconds and the child traces around the numeral and colours the balloons. The colouring-in takes perhaps 20 minutes or more. The exercise is really colouring in and not mathematics

Kevan Collins, Director of the Primary Strategy has recently emphasised that greater emphasis should be placed on teacher assessment of seven year olds (Slater, 2003, p.6): the value of children's own representations in mathematics could offer a key opportunity in supporting this.

Children's written mathematics in the classroom

The use of worksheets in mathematics appears to be widespread in England. Alexander's significant study of five nations – France, Russia, India, the USA and England – found that the teaching of mathematics worldwide is heavily influenced by textbooks and worksheets (Alexander. 2000). For example, in the USA Alexander noted the tension between schools wanting to move away from the dominance of standard textbooks, and the concern of the school board and the government of raising standards. Where change was noted, instead of using workbooks exclusively teachers created their own worksheets for children to record their mathematics set by the teacher. Comparing the Dutch approach 'REM' (Freudenthal, 1968) to that of Queensland in Australia, Zevenbergen raises concerns about the philosophy that underpins mathematics curricula in Australia and many other western countries. She argues that 'while there are tokenistic references made to children's informal understandings, these are not central to curriculum design' (Zevenbergen, 2002, p. 4). This finding is mirrored by our study. There is as Cook argues, 'little formal or widespread practitioner interest in re-assessing the appropriacy of … traditional teaching strategies' (Cook, 1996, p. 57).

Official guidance

Recent government guidance of the teaching of numeracy in England emphasises 'children will need to have plenty of experience of using their own individual ways of recording addition and subtraction activities before they begin to record more formally' (QCA, 1999, p.19). Further support for children's own mathematical graphic representations can be found in the *Curriculum Guidance for the Foundation Stage* for teachers of children 3 to 5 years of age. Teachers are recommended to promote confidence in children when they begin to record their mathematics: 'asking children to 'put something down on paper' about what they have done or have found out will allow them to choose how to record or whether to, for example, use a picture, some kind of tally or write a number' (QCA, 2000, pp. 71 – 2). Though neither document supports these statements with detailed explanations of the value of such recording, the guidance for teachers is clear. However, three years after the introduction of the Numeracy Strategy, this was clearly an area causing difficulties for teachers. In 2001, one of the key findings of the DfES report was that written methods were still not linked to mental calculation strategies, and that there was a largely 'uncritical dependence on published schemes and worksheets' (DfES, 2001). In Australia Clarke stresses that there should be 'less emphasis on formal recording and algorithms; and allowing a variety of recording styles' (2000, p.5). The key emphasis from a number of studies of effective numeracy teaching are 'a clear focus on concepts and thinking; and emphasis on valuing children's strategies and encouraging children to share strategies and solutions' (Doig et al, 2003. p.25).

An HMI report entitled *Teaching of Calculation in Primary Schools* argues that:

> 'Worksheets which provide no space for pupils' personal jottings or expanded methods of recording, or which simply require them to write answers in the boxes provided, are unhelpful. Teachers see commercial schemes as support for providing progression but, in some instances the worksheets limit pupil's opportunities to record their mathematical thinking in a manner which make sense for them and

which might support their oral explanations later' (DfES. 2002. 8. p.5).

IN 2002 Ofsted emphasised that 'pupils are less successful at using words and symbols to express a general rule and derive a formula. Pupils of all ages have too few opportunities to record mathematics in a variety of forms and a range of contexts, including problem solving. In number, pupils are not always supported well enough in the use of informal methods of calculation when they cannot cope successfully with standard written methods' (p.2).

Yet in contrast a year later, in a section entitled 'Independent work' a key Ofsted report comments, 'the use of worksheets is a common and *potentially effective way* of enabling the teacher to focus on a specific group of pupils, while the rest work independently' – (italics not in the original), (HMI, 2003). We are considerably surprised that Ofsted should include any support for the use of worksheets, particularly when comments in previous reports (above) have raised concern about their use.

For children, the value of informal jottings and personal methods are largely unrecognised, since 'few (teachers) see the use of jottings to record their thoughts as useful aids to calculation' (DfES 2002. 10. p5). This is comparable with Zevenbergen's (2002) concerns. This over-reliance on worksheets or commercial schemes continues in Year 2:

> 'many of these simply require an answer (often in a box) or some colouring in. No space is given for pupils' jottings or for them to show how they worked out the answer. Linked to this is an emphasis on recording at the expense of the explanation of methods. Pupils rarely use informal methods of recording or jottings' (DfES. 2002.13. p. 6).

There was also concern that 'older children too lack encouragement to explore ways of working out calculations in personal and meaningful ways which has resulted in 'pupils' anxiety to solve the problem and 'get it right' (which) often leads them to use immediately what they feel is the security of a standard written method rather than a range of strategies related to the nature of the task' (DfES. 2002. 30. p.9).

In 2004, QCA annual report on mathematics highlighted the need for training 'in pupils' recording of calculations and emphasised the need for clarity 'at both local and national level' when using terms such as 'jottings' and 'informal written methods' (QCA, 2004).

We have made strong arguments against the use of worksheets: we recognise that some will suggest that there is still a place for their use but argue that there is no such thing as a 'good' worksheet since what is significant are those aspects of learning that worksheets inhibit and prevent. The recent study *International Trends in Primary Education* notes that whilst teaching methods 'are not usually prescribed… teaching practice may be influenced by means of textbooks' (Le Métais, 2003, p.75). Thus, whilst recent government initiatives have impacted on many aspects of mathematics teaching, the influence of published materials remains strong and opportunities for children to make effective links between their mental and written methods remain largely unrealised.`

Bi-numeracy

In his book Hughes points out that for many learners mathematics is 'more like an unfamiliar foreign language' (1986, p.42): for many children mathematics can be an alien land. Discussing this idea, Hughes proposed that the difficulty is that children have to learn to *translate* between mathematical language and their everyday language. But there is another highly significant aspect of language that has been largely ignored in the debate about the difficulties children experience in translating from home to school mathematics – the way in which young children learning to speak a second language also learn the *written language* – and it is this perspective that we believe can make a huge contribution to the debate.

Halliday (1975) describes learning a first language as 'learning how to mean', a phrase that sits well with our emphasis on children making meaning through their mathematical graphics. John-Steiner reminds us that learners learning a second language draw on their 'internal meaning system while comprehending or producing language; they are increasingly able to comprehend, condense and store information, they start the process of weaving the two meaning systems together' (John-Steiner, 1985, pp. 357, 364 – 5).

Our thesis is that through gradually 'weaving' their informal marks and the standard symbols of mathematics, they come to understand the 'foreign language' of written school mathematics (Worthington & Carruthers, 2003). We term this 'bi-numeracy' since the children are become competent in two graphical systems (their personal, informal mathematical graphics and standard abstract mathematical symbols and calculations).

Conclusion

In our work with teachers we are aware that many teachers of children 3 – 8 years recognise that 'just as copy writing does not help children understand writing at a deep level, 'colouring in' is not art (Worthington, 2003). As Anning and Ring in their study of children's drawings emphasise, there is general confusion about appropriate pedagogy to support children's drawing and that 'the contribution of children to the process of creating meanings in drawings in scarcely acknowledged' (2004, p.124).

In mathematics it is also important that children are supported as they build their understanding of mathematical symbols on their own informal marks, rather than trace over or copy numerals or colour in worksheets' (Worthington, 2003). The numerous examples of children's mathematical graphics that we have analysed demonstrate children's capacity to explore, investigate, invent, adapt and understand mathematical symbols and their ability to construct meaningful written methods which can be 'read', just as children's emergent writing is also read. The 'handwriting' and 'grammar' of mathematics develops in hand with the content and the mathematics itself. Ivanov emphasised 'that the most important thing is not the sign, but the whole utterance into whose composition the sign enters (1974, p. 237): the children with whom we have worked not only have a growing understanding of the meaning of the symbols but of the mathematical contexts in which they belong.

What was remarkable from the 500 examples of the use of blank paper was the almost total absence of children exploring their own ways of thinking and working out. In almost a thousand examples of children's written mathematics on worksheets and on blank paper from the teachers in our study, it is difficult to see the sort of purposeful and rich mathematical activities that 'help children see themselves as mathematicians' (QCA, 2000, p. 70-71). Andrews, Ball et al (2004), highlight the non-statutory guidance of the National Curriculum Council (1989), who emphasised that mathematics should offer pupils 'intellectual excitement'. Reading the numerous examples given by teachers in our study, opportunities for intellectual excitement in mathematics appear to be largely absent. Straker and Coulson have also identified the 'need to help Primary teachers to develop a deeper understanding of the maths they teach so they can raise their expectations of children' (2004). In their proposals for revising the framework they intend to tackle the issue of the difficulties children experience with calculation methods. It is vital that the children have appropriate support to build on their own informal methods from the Foundation stage: focusing on young children's mathematical graphics will give a grounding for real mathematical thinking and create intellectual continuity through to Key stage 2.

It is clear is that teachers of young children urgently need a new vision to support children's written methods in mathematics: we believe that our research can contribute to this vision and understanding.

Why encourage children's mathematical graphics?
This is, we have argued, 'provocative maths'; that is to say it inspires, motivates and challenges children's minds. It requires them to 'gradually make existing perceptions explicit, to try out alternative ways of thinking, looking and representing' (Worthington & Carruthers, 1998, p.15). Our central thesis is that the gap in children's mathematical understanding is bridged through supporting their intuitive mathematical graphics.

From our study the use of worksheets in mathematics appears prolific, yet clearly there are many who question their relevance in supporting young learners. Our research into children's mathematical graphics (Worthington & Carruthers, 2003a) shows that children can only become bi-numerate when they are able to make connections between their informal and intuitive marks within contexts that are personally meaningful. This takes time: making sense of the written symbolism of mathematics is part of the young child's continuing search for meaning through diverse symbol systems and ways of representing their worlds. Children must be supported in their own meaning-construction rather than colouring-in ours (Worthington & Carruthers, 2003b).

What does it mean to encourage children to 'see themselves as mathematicians' (QCA, 2000)? We argue that a bi-numerate approach supports what Holton et al (2001) term 'mathematical play' (Carruthers & Worthington, 2004. p.33). They emphasise that 'in the context of written calculations… 'mathematical play' with ideas and processes seems a natural strategy, undertaken by young children and research mathematicians alike' (p. 413). Thompson proposes that teachers must ensure that 'substantial emphasis (is) given to mental methods *and to the development of children's own methods of recording at Key Stage 1'* (our italics), (p. 106, 13). However, as our research shows, in practice this link is seldom able to be made by children in Foundation and Key Stage 1.

Teachers have a vital role in the process of bi-numeracy since it is they who can help the children make connections. We will never see the demise of worksheets in mathematics teaching if practitioners fail to support children's own ways of exploring their mathematical thinking on paper. Change can only thrive if 'teachers, parents and others who have influence and foresight support it at the grass roots to help children become confident mathematicians' (Carruthers, 1998, p.17).

Key Recommendations

Review of the Curriculum Guidance for the Foundation Stage and the National Numeracy Strategy guidance:

- to clarify the position of early 'written' mathematics and to extend the present guidance to more fully support teachers of young children
- to provide professional development materials focusing on this key aspect of mathematics
- to provide demonstration of the pedagogical approaches needed to support teachers through professional development courses

We would emphasise that these aspects will need time for reflection so that new materials and training are not 'add-ons' but can genuinely make a difference. This would then link to 'Birth to Three Matters' which emphasises the importance of 'communicating meaning' and 'creating and experimenting with one's own marks and symbols' (2002, p.6/7).

Professional development of teachers:

- to support understanding of children's own mathematical graphics
- develop pedagogical approaches that support children's own mathematical graphics

Initial Teacher Training:

- ensure that the development of children's early mathematical graphics and own written methods and the pedagogy to support this is included in future modules for teachers of the Foundation Stage and Key Stage 1.

This point is integral to a child development perspective which has largely been missing from initial teacher training in recent years and significantly connects with all children's forms of meaning making through visual representation, including art, early writing and multi-modal representations (e.g. Kress, 1997).

Assessment:

- Review of the Key Stage SATs so that children's own mathematical graphics are valued in terms of the thinking they show
- Training needs to be provided for Ofsted and HMI so that they understand the significance of young children's mathematical graphics and the wider context of visual representation

When they are introduced to this paradigm and to examples of children's mathematical graphics, students' and teachers' responses are almost always of surprise and great interest – that it makes sense to encourage children's own mathematical marks and written methods, that working in this way offers a real alternative to the use of worksheets and above all that it offers tremendous insights into children's understanding and development. But the benefits are greatest for the children.

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