

## **Response from the Children's Mathematics Network to the consultation for the National Numeracy Strategy: a need for understanding children's early written mathematics**

### **Background**

**The Children's Mathematics Network (CMN)** focuses on children's own mathematical graphics and their meanings: these support children's deep levels of understanding of the abstract symbols of mathematics and all areas of 'written' mathematics including calculations.

The network is a non-profit-making organization for teachers, practitioners, students, researchers and teacher educators working with children in the birth – 8 year age range. It is a grassroots network, with children and teachers at the heart of it.

The Network supports the guidance for the Foundation Stage and the National Numeracy Strategy. It was established in 2003 and represents teachers and educators throughout England.

Members were invited to submit their comments on the revision of the National Numeracy documents.

### **RECOMMENDATIONS**

**The Children's Mathematics Network recommends the following for revision to the National Numeracy Strategy:**

- **Children's *cognitive development* needs to be given higher priority in curriculum documents**
- **Teachers need greater support to understand the significance of *child-initiated play, talk and thinking for mathematics* – to see beyond the resources**
- **Teachers need guidance to understand that *'practical' mathematics* alone cannot help children build understanding of the abstract symbols of mathematics or early written mathematics**
- ***Official guidance* needs to be much clearer about the teaching of 'written' mathematics in the Foundation stage and Key stage 1. Emphasis need to be particularly given that the value of mark-making goes beyond writing and has a highly significant role in supporting the understanding of abstract mathematical symbols and all written mathematics (including calculations)**
- **Teachers need support and training in understanding the *development* of children's mathematical graphics**
- **Teachers need support and training in *teaching* mathematical graphics**

- **A range of exemplars of children's mathematical graphics to show development from their earliest marks and to indicate progression will support teachers' and practitioners' assessment.**
- **Addition of key objective followed through from Foundation stage through Key Stage one:**
  - **Develop meaning through their own mathematical graphics and other multi-modal forms**

CMN would like to emphasise the importance of continuity throughout the birth to eight age-range: the following details include reference to recent research on the teaching of written mathematics; *Birth to Three Matters; the Curriculum Guidance for the Foundation Stage*, in addition to guidance for the *National Numeracy Strategy*.

### **Research overview**

There is now well documented research that children can express their mathematical thinking and build meaning through their own mathematical graphics on paper at a very early age, (i.e. Hughes, 1986; Munn, 1995; Worthington and Carruthers, 2003a). If children's own mathematical graphics are supported, nurtured and most of all understood by teachers, then when children begin to take on standard written methods of mathematics they will understand and be able to better use and apply these methods.

Currently official guidance on the pedagogy that supports the children's own marks is unclear and many practitioners remain confused (Worthington & Carruthers, 2003b). Recent research by Carruthers and Worthington (2005) is particularly useful since they trace the development of children's mathematical graphics from two years of age until the end of key stage one. They are also developing research into pedagogical approaches that support children's mathematical understanding.

*See also p. 6 - 10 of this document for summary details of two studies with teachers and practitioners.*

**'We want to align the two frameworks more closely'** (DfES, 2005. Draft. p. 2.).

The Children's Mathematics Network would like to emphasise the importance of *Birth to Three Matters* in this:

- A Competent learner
- **Representing**
- **Responding to the world with marks and symbols**
- A skilful communicator
- **Making meaning**
- **Communicating meaning**

From the official guidance above there are direct links to mathematical graphics (in bold above): CMN would like to propose that the links to 'making meaning' be made clear by emphasising its importance through the CGFS, and linking this to the *Guidance for Teaching Written Methods* in the National Numeracy Strategy. There is a really important thread here from birth through Key Stage one.

### **Proposed changes within the 'Early Years Foundation Stage' curriculum**

CMN support the proposed change from 'Mathematics' to 'Problem-solving, reasoning and numeracy' (as in the Direction of Travel document, 2005) for the new *Early Years Foundation Stage*: as Oers argues, 'mathematics as a subject is really a matter of problem solving with symbolic tools (2001, p. 63).

**'Young children's learning is not compartmentalised. They learn when they make connections between experiences that are related to any aspect of their life in the setting, at home and in the community' (QCA, 2000).**

The statements below from the current Foundation Stage Curriculum support children's own mathematical graphics (early written mathematics) in the Foundation Stage, including some from *Communication Language and Literacy* and some from *Creative Development*. The Children's Mathematics Network recommend that these connections to mathematics also be made explicit

### **Communication, language and literacy**

- As language develops, thought becomes less dependent on action (p.45)
- They can capture their actions in drawing, early writing or painting, and retell events to friends, practitioners and parents. They are learning that pictures and words are symbolic ways of preserving meaning (p.45)
- Children are more likely to write as part of purposeful play (p.46)
- Children will learn about the different purposes of writing by seeing practitioners write for real purposes (p. 46)
- Children's experience of different scripts at home should be acknowledge and built on when learning (p.47)
- Children's experiences of different scripts at home should be acknowledged and built (p.47)
- Ask children to give reasons, further explanations or evidence for what they say (p.59)
- Take an interest in what and how children think and not just what they know (p.59)
- Encourage children to explore and ask about meanings (p.59)

- Children should attempt writing for different purposes, using different forms – and ascribe meanings to marks (P.64)

## Mathematical development

- It is important that children's experiences of mathematics are enjoyable and meaningful and that their confidence is always fostered through building from what they know, understand and are able to do. (p.71)
- Asking children to 'put something on paper' about what they have done or found out will allow them to choose how to record or whether to, for example, use a picture, some kind of tally or write a number. Children are most likely to want to 'put something down' when the record has some purpose for them. (p.71)
- It will provide a much stronger foundation for mathematical development than an introduction to 'sums' such as ' $2 = 3 =$ ' before children have had opportunities to explore number situations, count, begin to calculate, talk about how they worked things out and record in their own ways. (p.72)

## Creative development

- Children will learn to respond, explore, express, communicate their ideas and use their imagination when they have sufficient time to explore ... experiment with ideas and bring their own ideas to the process. (p.117)
- Children need time to take risks, make mistakes and be adventurous... if children are to have the confidence to try innovative ideas, they must be given time to work at their own pace. (p.117)
- (Creativity) begins with curiosity and involves children in exploration and experimentation. As they express their creativity, they draw upon their imagination and originality. (p.118)
- Children's creativity develops over time and takes time. It is best facilitated by adults who sensitively support this process and do not dominate it. If they are to be truly creative, children need the freedom to develop their own ideas and the support of adults who can help them gain the skills that enable their creativity to have expression. (p.118)
- Creativity is not about pleasing adults or producing adult-determined art. (p. 118)
- The practitioner must create a climate where curiosity is encouraged and where children can experience the unexpected. (p.1018)

**All the above areas of learning are connected to children's mathematical graphics: however, often teachers and practitioners do not see the connections between one area of learning and another, for example creativity connected with mathematics. The CMN recommends that these connections be made more explicit.**

The emphasis on continuity between Foundation and Key stage one is vital in all areas. The *Continuity* INSET materials (QCA, 2005) stress the importance of developing a smooth transition to key stage one from the Foundation stage and of continuing a familiar play based approach to

learning for young children as they move from one stage to the other. However, there also needs to be greater emphasis on children's development and what it looks like as understanding and skills grow. This includes a need for continuity of children's own mathematical graphics in order to support deep levels of understanding of abstract written symbols of mathematics and all aspects of written mathematics, including calculations.

Guidance on appropriate pedagogy to support children in this significant area of their development and provide continuity of experience also needs to be provided.

**The Children's Mathematics Network recommends that greater emphasis needs to be given to the significance of children building *mathematical meanings* through using their own marks.**

## **Teaching Written Calculations: official guidance from the National Numeracy Strategy**

The statements below support children's own mathematical graphics (early written mathematics in the Reception & KS1): however since there is no clear guidance on this aspect of the curriculum, many teachers are confused: 'There seems to be a 'no mans land' between practical maths and the standard written calculations expected in schools' (Nursery Teacher, Cambridge, 2006).

Even since the introduction of the Numeracy Strategy in 1999 there is still a heavy reliance on worksheets for mathematics instead of supporting children's own thinking on blank paper. The abundant use of worksheets for mathematics will continue to dominate in Foundation and Key Stage 1 unless teachers have a clear understanding of the value of children's own mathematical graphics (*see p. of this document for research findings*). As Ian Sugarman (1995) argues (referring to published schemas and worksheets), 'more significantly there is a strong sense that teaching arithmetic is all about showing children what to do rather than how to think'.

### **Guidance from QCA (1999) *Teaching Written Calculations*, London: QCA:**

- Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to develop new ideas. Written recording helps children to clarify their thinking. As children's mental methods become more sophisticated, they will be able to use written methods that are more concise and more structured (p.11)
- Children should be encouraged to see mathematics as a written as well as a spoken language (p.11)
- At first, children's recording may not be easy for someone else to interpret, but they form an important stage in developing fluency (p.12)
- Children attempting to use formal written methods without a secure understanding will try to remember rules, which may result in unnecessary mistaken applications of the standard method (p.12)
- As children gain in understanding, their written methods will become more fluent and more efficient (p.18)

- Children will need to have plenty of experience of using their own individual ways of recording addition and subtraction activities before they begin to record more formally (p.19)
- A mixture of words and symbols will be used by children in order to explain to someone else the mental methods they have used. Children will use a variety of ways of recording addition and subtraction reflecting the variety of mental methods used (p.20)
- Different methods are often used for the same calculation and this can lead to useful discussions (p.21)
- Standard written methods will be developed progressively, but it is most important that at all stages children understand the methods they are using (p.49)

Whilst we believe that these statements are vital, the documents provide insufficient information for teachers and all early years' practitioners regarding children's early development and appropriate pedagogical approaches.

There is also a very real need for appropriate exemplars of children's mathematical graphics to be included in any curriculum guidance for teachers and practitioners.

---

**'We want the Frameworks to support schools to: enrich learning; develop curricular, pedagogic and subject knowledge; continue to raise the expectations of children, teachers and parents', (DfES, 2005. Draft. p. 1.).**

## National Numeracy Strategy

**The Children's Mathematics Network proposes that an additional key objective be included in the framework.**

### **Additional Key Objective recommended:**

- **Develop meaning through children's mathematical graphics and other multi-modal forms**

**Note:** term 'children's mathematical graphics' - developed by Worthington & Carruthers, 2003a, & Carruthers & Worthington, 2005, (see end of this document for full references).

**Recent research** with children from 2 – 8 years has shown that they can develop a deep understanding of the abstract written symbols of mathematics, of calculations and all aspects of 'written' mathematics (e.g. data handling) if they are supported in building on their earliest marks as they construct mathematical meaning, (Carruthers & Worthington, 2005; Worthington & Carruthers, 2005; Worthington & Carruthers, 2003a.). However, there is confusion about teaching the beginnings of 'written' mathematics' (see research findings, below).

This research is supported by the Foundation Stage document which recommends: **'Early Years experience should build on what children already know and can do'** (CGfFS, Principles for Early Years education, 2000, p.11). However, evidence from the two research

projects summarized below, highlight some of the difficulties teachers have in providing experiences for children that build on their earliest understandings of written marks and symbols.

**‘Assessment of children’s progress is purposeful, straightforward and informative’,** (DfES, 2005. Draft: p.1.).

- **Assessment** needs to be built on sound knowledge of young children’s development of mathematical graphics so that they are valued in terms of children’s meanings and the thinking they show
- **Children’s development** in understanding standard abstract symbols and early calculations is complex and needs to be reflected in guidance for teachers and greater support for teachers
- It is important that **examples of young children’s mathematical graphics and own written methods be drawn from published research** of children’s mathematics in the classroom. For example the document *Teaching Written Calculations* (QCA 1990, p. 20) in particular are very much adult-directed and are not children’s own examples, but teachers’ written examples for children to copy. Recent published research on this development can support understanding of children’s development, assessment and pedagogy of children’s mathematical graphics (early ‘written’ mathematics)

## 1. Research with teachers on early ‘written’ mathematics

**This research explored the extent to which recommendations in the National Numeracy Strategy had influenced classroom practice, in respect of children’s own marks and written methods – teachers (2) were asked:**

- Do you give children worksheets for mathematics?
- Do you give children blank paper for mathematics?

Teachers were also asked to provide examples of the sort of things that children might do on blank paper (if they had opportunities to use it).

Responses were gathered over a one-year period from 273 teachers in four areas of England: three were large cities: one in the north of England, one in the west and one in the south-west. The remaining area was a large, rural county (Worthington & Carruthers, 2003a; Worthington & Carruthers, 2003b).

**Note 2:** *the term ‘teachers’ is used for brevity here, although the research included a range of early years practitioners*

### Some findings

- The large difference in the use of worksheets provided in some settings was evident
- Different types of Early Years settings and classes with different ages of children highlighted different levels of worksheet use

**Table 1.1**

<i>Type of setting</i>	<i>Percentage using worksheets</i>
------------------------	------------------------------------

Maintained nursery classes	20
Private day nurseries	63
Pre-schools (voluntary)	72
School - Reception classes	89
School - Reception / Y1 combined	100
School – Y1	100
School - Y2	100
School – Y3	100

- Whilst 79% of teachers of children 3 – 8 years used worksheets, 82% of the same teachers also either allowed or encouraged some use of blank paper
- However, almost 85% of examples they gave were either when the teacher told the child what to do or how to record, or when the teacher produced what was in effect, a copy of a worksheet
- 9% of examples referred to the use of blank paper within role play: however role play was at the time rarely provided beyond Reception classes (and in 2006 remains the exception in Y1 classes, in spite of recommendations in the *Continuity* INSET materials, QCA, 2005)
- In only 6% of examples of mathematics on blank paper, did teachers refer to children 'making their own marks' or 'making jottings', painting a disappointing picture since most of these teachers used worksheets for a greater part of the time
- 77% of teachers responded that children took their mathematical marks home: of the 6% of teachers whose children did occasionally use blank paper for mathematics, fewer than a quarter kept copies (or originals) for school records – and only a tiny sample of what children had done, was saved.
- Blank paper was more likely to be used only occasionally by those who used it at all, and therefore its use had a different status to worksheets
- Marks made by children during play were not seen as significant in contributing to children's developing understanding

**In mathematics it appears that children's own mathematical marks on paper are seen as significant only when they are the outcome of a teacher- directed activity. These findings point to written mathematics that is largely on worksheets or following the direction of the teacher.**

**Furthermore, it must be almost impossible to trace children's development and to assess their understanding unless children's own representations are dated, annotated and kept: valuable assessment opportunities are lost.**

**For the children, valuable learning opportunities are closed.**

## **2. Recent research with teachers on creativity in mathematics**

**This research explored what teachers knew about teaching early 'written' mathematics from official guidance. It also explored the extent to which teachers view mathematics as**

providing opportunities for creative thinking and ways of working in mathematics.

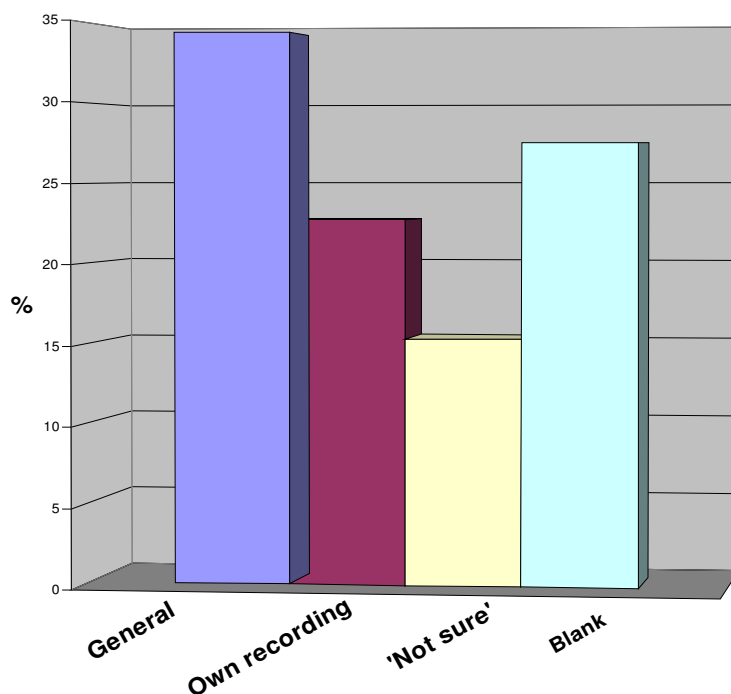
What did they know of any official guidance on teaching early 'written' mathematics? What sort of things had they see children do that they identified as 'creative'? How did their understanding of early childhood development – particularly relating to play, talk, thinking and mark-making to support children's learning – inform their practice?

This study further explored some issues from the 2003 study.

- Research based on questionnaires from 231 teachers and practitioners in three areas of England and subsequently follow-up telephone interviews were conducted with 10% of teachers to explore issues in greater depth
- Responses were from maintained nurseries, voluntary pre-schools, private day nurseries, Reception classes, combined Reception and Y1 classes and Y2. classes (Worthington & Carruthers, 2005).

See table 2.1 below

**Knowledge about Foundation Curriculum - teaching 'written' mathematics to young children**



**Table 2.1: What does official guidance suggest about the teaching of early written mathematics?**

**Some findings – knowledge of official guidance on teaching written mathematics (see Table 2.1 above)**

- 35% of teachers believed that official guidance on teaching written mathematics, was to provide a range of resources and practical activities (see 'general' comments in table 2.1)
- Only 23% of teachers recognised that official guidance recommended children's 'own recording'
- Almost 43% of teachers said they were either unsure or confused by the official guidance – or left this question blank

### **Teachers' perceptions of creativity in mathematics**

- Teachers cited only very occasional opportunities when children used their own marks to explore mathematical meanings
- Examples cited suggest that teachers tend to see creativity in mathematics as concerned with *specific resources or activities that adults planned and provided*, rather than learning *processes* in which children were engaged
- Teachers were invited to provide specific examples of something they had seen a child do that they regarded as creative in mathematics: however, only *one* teacher out of 231 responded to this question, suggesting that teachers fail to 'see' mathematics when observing children
- Mathematics through self-initiated play, talk and thinking were cited as creative by only 9% of teachers
- Just 5% of teachers cited children's mathematical marks (*children's own mathematical graphics* or early written mathematics), as creative
- Responses showed that children's own mathematical marks were not kept by teachers to provide information of their development or used for assessment purposes

In the course of three years between the two studies, teachers remain confused and unclear about how they can teach early written mathematics

## **RECOMMENDATIONS**

**The Children's Mathematics Network recommends the following for revision to the National Numeracy Strategy:**

- **Children's *cognitive development* needs to be given higher priority in curriculum documents**

- Teachers need greater support to understand the significance of *child-initiated play, talk and thinking for mathematics* – to see beyond the resources
- Teachers need guidance to understand that *‘practical’ mathematics* alone cannot help children build understanding of the abstract symbols of mathematics or early written mathematics
- *Official guidance* needs to be much clearer about the teaching of ‘written’ mathematics in the Foundation stage and Key stage 1. Emphasis need to be particularly given that the value of mark-making goes beyond writing and has a highly significant role in supporting the understanding of abstract mathematical symbols and all written mathematics (including calculations)
- Teachers need support and training in understanding the *development* of children’s mathematical graphics
- Teachers need support and training in *teaching* mathematical graphics
- *A range of exemplars* of children’s mathematical graphics to show development from their earliest marks and to indicate progression will support teachers’ and practitioners’ assessment.
- Addition of key objective followed through from Foundation stage through Key Stage one:
  - Develop meaning through their own mathematical graphics and other multi-modal forms

---

#### References:

Carruthers, E. & Worthington, M. (2005) ‘Making sense of mathematical graphics: the development of understanding abstract symbolism’, *European Early Childhood Education Research Journal* (EECERA) Volume 13, No. 1 2005.

Hughes, M. (1986) *Children and Number: Difficulties in Learning Mathematics*, Oxford. Blackwell.

Munn, P. (2005). 'Writing and Number', in *Teaching and learning Early Number*, Editor: Ian Thompson. Buckingham: Open University Press.

Oers, B. van. (2001a) 'Educational forms of intuition in mathematical culture', in *Educational Studies in Mathematics*, **46**: 59-85, 2001

QCA (1999) *Teaching Written Calculations*, London: QCA

QCA (2005) *Continuing the learning Journey*, London: QCA.

Sugarman, Ian. (2005) 'Teaching for strategies', in *Teaching and Learning Early Number*, Ian Thompson, (Ed). Buckingham: Open University Press.

Worthington, M & Carruthers, E. (2003a) *Children's Mathematics: Making Marks, Making Meaning*, London: Paul Chapman Publishing, (second edition, July 2006)

Worthington, M. & Carruthers, E. (2003b) 'Teachers' practices concerning children's early 'written' mathematics', paper presented at the *European Early Childhood Education Research (EECERA)* conference, University of Strathclyde, 2003.

Worthington, M., & Carruthers, E. (2005) 'Creativity and Cognition: the Art of Children's Mathematics': paper presented at Roehampton *Art in Early Childhood* conference, July, 2005.

---

**E-mail:** [info@childrens-mathematics.net](mailto:info@childrens-mathematics.net)