

# Supporting children's calculations and written mathematics Official Guidance (England)

## **QCA. (2000) *Curriculum Guidance for the Foundation Stage.***

'Young children's learning is not compartmentalised. They learn when they make connections between experiences that are related to any aspect of their life in the setting, at home and in the community' (p.45)

## **EYFS: (2007) *Effective Practice: Play and Exploration*, London: DfES.**

Page numbers for the EYFS refer to documents online: <http://www.standards.dfes.gov.uk/eyfs/>

### **The importance of play for learning in the EYFS**

'Play helps young children to be competent learners who can make connections and who can create and transform ideas and knowledge, because they are imaginative and expressive' (p.2)

'Children engaged in play and exploration are learning through experience, because young children's development and learning, whether physical, social, emotional, moral or cognitive, requires real, hands-on engagement – it cannot be done by means of a worksheet.' (p. 4)

'Each early years setting is a dispositional milieu: a place that presents children with particular views of what it means to learn and be a learner. Children will fit in and thrive in a milieu, or they may avoid, resist or try to change that milieu. The type of learners children become will be influenced by the environment. Effective dispositional milieu are said to be characterized by challenging activities that invite involvement, provision for children to make choices and share responsibility – meaning that children are powerful, competent members of a learning community of players who share ideas and roles. Further, effective dispositional milieu celebrates diversity, and values different cultural approaches to play. The practitioners are flexible and go with the flow of children's play, delighting in spontaneity but still able to recognize and reflect on what the children have achieved,' (p. 7).

## **Meanings, marks and mathematics in the EYFS**

### **1. Problem Solving, Reasoning and Numeracy**

#### **DfES (2007) *Learning and Development: Problem Solving, Reasoning and Numeracy*: EYFS.**

- **Value the children's own graphics** and practical explorations of Problem Solving, Reasoning and Numeracy (p.2.)

*(Continued overleaf...)*

## **DfES (2007) *Problem Solving, Reasoning and Numeracy: Numbers as Labels and for Counting, EYFS.***

### **22 – 36 months:**

- (children) create and experiment with symbols and marks (p.2)
- (Look, listen and note): the contexts in which young children make marks and symbols (p.2)
- (Effective practice): talk about how the symbols and marks you make stand for numbers and for quantities (p.2)
- (Planning and resourcing): introduce number labels to use outdoors for car number plates, house and bus number (p.2)
- (Planning and resourcing): create a 'number rich' environment in the home play area. Introduce numbers as they are used at home, by having a clock, a telephone and a washing machine (p. 2)

### **40- 60+ months:**

- (children) recognise some numerals of personal significance (p. 3)
- (children) begin to represent numbers using fingers, marks on paper or pictures (p.4)
- (children) use developing mathematical ideas and methods to solve practical problems (p. 4/5)
- (Look, listen and note): how children begin to represent numbers using fingers, marks on paper or pictures (p.4)
- (Effective practice): ensure children are involved in making displays (p.4)
- (Effective practice): add numerals to all areas of the curriculum (p.4)
- (Effective practice): make books about numbers that have meaning for the child such as favourite numbers, birth dates or telephone numbers (p.4)
- (Planning and resourcing): display numerals in purposeful contexts (p.3)

## **DfES (2007) *Problem Solving, Reasoning and Numeracy: Calculating, EYFS.***

### **30 – 50 months:**

- (children) separate a group of three or four objects in different ways (p.2)
- (children) use own methods to work through a problem (p.4)
- (Look, listen and note): the strategies children use (p.2)

### **40-60 months**

- (Children) use own methods to work through a problem (p.2)
- (Look, listen and note): methods children have used to answer a problem they have posed (p.2)
- (Effective practice): show an interest in how children solve problems and value their different solutions (p.2)
- (Planning and resourcing): encourage children to record what they have done, for example by drawing or tallying (p.2)
- (Look, listen and note): the variety of responses when children work out a calculation (p.3)
- (Look, listen and note): children working out what remains if something is taken away (p.3)
- (Effective practice): discuss with children how problems relate to one another they have met, and their different solutions (p.3)
- (Effective practice): Encourage children to extend problems (p.2)
- (Effective practice): use mathematical vocabulary and demonstrate methods of recording, using standard notation where appropriate (p. 3)
- (Planning and resourcing): provide a range of number resources and encourage children to be creative in thinking up problems and solutions in all areas of learning (p.3)

## **DfES (2007) *Problem Solving, Reasoning and Numeracy: Look, Listen and Note, EYFS.***

### **Numbers as Labels and for Counting**

#### **22 – 36 months:**

- How young children show their understanding of number labels such as 1, 2, 3, (p.1)
- The contexts in which young children use marks and symbols (p.1)
- Situations that prompt children to talk about numbers (p.1)

#### **30 – 50 months:**

- The strategies that children use to match number and quantity, for example, by using fingers or tallying or by making marks (p.2)

#### **40- 60+ months:**

- The personal numbers that children refer to, such as their age, house number, telephone number or how many people in their family (p.2)
- Instances of children counting an irregular arrangement of objects (p.2)
- Children's methods of counting (p.2)
- How children begin to represent numbers using fingers, marks on paper or pictures (p.2)
- Children's recognition of numerals (p.2)
- How children use their developing understanding of maths to solve mathematical problems (p.2)

### **Calculating**

#### **40- 60+ months:**

- The methods children use to answer a problem they have posed (p.2)
- How children find the sum of two numbers (p.2)
- The variety of responses when children work out a calculation (p.2)
- The ways children count represented groups of the same size (p.2)
- How children share objects (p.2)
- Children working out what remains if something is taken away (p.2)
- Create a 'number-rich environment in the home play area (p.2)

## **2. Communication, Language and Literacy**

### **DfES (2007) *Communication, Language and Literacy: Writing, EYFS.***

#### **22- 36 months:**

- (Children) distinguish between the different marks they make (p.1)
- (Look, listen and note): what children tell you about the marks they make (p.1)
- (Effective practice): draw attention to marks, signs and symbols in the environment and talk about what they represent. Ensure this involves recognition of English and other scripts (p.1)
- (Planning and resourcing): Provide materials which reflect a cultural spread, so that children see symbols and marks with which they are familiar (p.1)

#### **30 – 50 months:**

- (Children) sometimes give meaning to marks as they draw and paint (p.2)
- (Children) ascribe meanings to the marks that they use in different places (p.2)

- (Look, listen and note): the marks children make and the meanings they give to them (p.2)
- (Planning and resourcing): include opportunities for writing during role-play and other activities (p.2)

#### **40 – 60+ months**

- (Children) use writing as a means of recording and communicating (p.2)
- (Look, listen and note): how children use writing to record things or communicate (p.2)
- (Planning and resourcing): provide materials and opportunities for children to use writing in their play, and create opportunities for independent and group writing (p.2)

### ***DfES (2007) Communication, Language and Literacy: Language for Thinking, EYFS.***

#### **30 – 50 months:**

- (Children) use talk to give new meanings to objects and actions, treating them as symbols for other things (p.2)
- (Look, listen and note): how children use talk to connect ideas and explain things (p.2)

#### **40 – 60+ months:**

- (Children) use talk to organise, sequence and clarify their thinking, ideas, feelings and events (p. 3)
- (Look, listen and note): how children use talk to reflect upon, clarify, sequence and think about experiences, ideas and feelings (p.2)
- (Effective practice): take an interest in what and how children think and not just what they know (P.3)
- (Planning and resourcing): provide for, initiate and join in imaginative play and role play, encouraging children to talk about what is happening (p.3/4)
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### ***DfES (2007) Communication, Language and Literacy: Language for Thinking, EYFS.***

#### **22 – 36 months:**

- (Children) begin to use representation as a form of communication
- (Look, listen and note): signs, body language and gestures that young children use in response to their experiences (p.1)
- (Effective practice): help children value their creative responses (p.1)

#### **30 – 50 months:**

- (Children) use language and other forms of communication to share the things they create (p.2)
- (Children) capture experiences and responses with music, dance, paint and other materials or words (p.2)
- (Look, listen and note): the ways children capture their experiences (p.2)
- (Effective practice): provide appropriate materials and extend children's thinking through involvement in their play, using questions thoughtfully and appropriately (p.2)
- (Planning and resourcing): ensure that there is enough time for children to express their ideas, thoughts and feelings in a variety of ways (p.2)

#### **40 – 60+ months**

- (Children) talk about personal intentions, describing what they were trying to do (p.2)
- (Children) respond to comments and questions, entering into dialogue about their creations (p.2)
- (Children) make comparisons and create new connections (p.2)
- (Children) express and communicate their ideas, thoughts and feelings (p.2)
- (Look, listen and note): the connections children make as they respond to their various experiences

- (Effective practice): support children in expressing opinions (p.2)
  - (Effective practice): be alert to children's changing interest and the ways they respond to experiences (p.2)
  - (Planning and resourcing): introduce language that enables children to talk about their experiences in greater detail (p.2)
  - (Planning and resourcing): provide children with examples of how other people have responded to experiences, engage them in discussions of these examples and help them make connections
  - (Planning and resourcing): provide and organise resources and materials so children can make their own choices to express their ideas (p.2)
  - (Planning and resourcing): be sensitive to the needs of children who may now be able to express themselves in English (p.2/3)
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### **3. Creative Development: Being Creative - Responding to Experiences, Expressing and Communicating Ideas**

#### **22 – 36 months:**

- (Children) seek to make sense of what they see, hear, smell, touch and feel (p.1)
- (Children) begin to use representation as a form of communication (p.1)
- (Look, listen and note): word plays, signs, body language and gestures that young children use (p.1)
- (Effective practice): help children value their creative responses by your interest in the way they move or represent (p.1)

#### **30 – 50 months:**

- (Children) use language and other forms of communication to share the things they create (p.2)
- (Effective practice): be interested in children's responses, observing their actions and listening carefully (p.2)

#### **40 – 60+ months**

- (Children) talk about personal intentions, describing what they were trying to do (p.2)
  - (Children) respond to comments and questions, entering into dialogue about their creations (p.2)
  - (Children) express and communicate their ideas, thoughts and feelings (p.2)
  - (Look, listen and note): the connections children make as they respond to different experiences (p.2)
  - (Planning and resourcing): introduce language that enables children to talk about their experiences in greater depth and detail (p.2)
  - (Planning and resourcing): provide children with examples of how other people have responded to experiences, engage them in discussions of these examples and help them to make links and connections (p.2)
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## **4. Assessment**

### **DfES (2007) *Effective Practice: Observation, Assessment and Planning, EYFS.***

We need to ask ourselves: what does our observation and any other evidence of learning we have collected (such as examples of children's mark-making...) tell us about the child's learning and development? What was new – something we had not observed before? When we do this regularly we have evidence of the children's progress over time and we gain insights into the children's learning, development and their needs.

Effective assessment involves evaluation or decisions about the child's progress and their learning and development needs and gives us the information we need to plan for the next steps. This is called assessment for learning: it is the formative assessment, based on observations, which informs or guides everyday planning.

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## Primary National Strategy: Numeracy

### QCA (1999) *Teaching Written Calculation*

- Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to develop new ideas. Written recording helps children to clarify their thinking. As children's mental methods become more sophisticated, they will be able to use written methods that are more concise and more structured (p.11)
- Children should be encouraged to see mathematics as a written as well as a spoken language (p.11)
- At first, children's recording may not be easy for someone else to interpret, but they form an important stage in developing fluency (p.12)
- Children attempting to use formal written methods without a secure understanding will try to remember rules, which may result in unnecessary mistaken applications of the standard method (p.12)
- As children gain in understanding, their written methods will become more fluent and more efficient (p.18)
- Children will need to have plenty of experience of using their own individual ways of recording addition and subtraction activities before they begin to record more formally (p.19)
- A mixture of words and symbols will be used by children in order to explain to someone else the mental methods they have used. Children will use a variety of ways of recording addition and subtraction reflecting the variety of mental methods used (p.20)
- Different methods are often used for the same calculation and this can lead to useful discussions (p.21)
- Standard written methods will be developed progressively, but it is most important that at all stages children understand the methods they are using (p.49)

### A 'new approach': The Primary Strategy

'We want a new approach that will help more schools and teachers to develop the detailed understanding and confidence to apply the principles of good learning and teaching across the whole curriculum, in a flexible way' (p.29) DfES (2003) *Excellence and Enjoyment*.

(Continued overleaf...)

## The Renewed Primary Framework

Page numbers refer to documents online:

<http://www.standards.dfes.gov.uk/primaryframeworks/mathematics/Papers/Calculation/>

### Calculation

- As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally, (p.1).
- When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy.

### Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts, (p.2).

#### Year 1: Calculating

- Relate addition to counting on; recognise that addition can be done in any order; use practical and informal written methods to support the addition of a one-digit number or a multiple of 10 to a one-digit or two-digit number
- Understand subtraction as 'take away' and find a 'difference' by counting up; use practical and informal written methods to support the subtraction of a one-digit number from a one-digit or two-digit number and a multiple of 10 from a two-digit number
- Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences

#### Year 2: Using and Applying Mathematics

- Solve problems involving addition, subtraction, multiplication or division in contexts of numbers, measures or pounds and pence

- Identify and record the information or calculation needed to solve a puzzle or problem; carry out the steps or calculations and check the solution in the context of the problem

## Year 2: Calculations

- Use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders

## Embedding Using and Applying Mathematics in all strands

- To be successful, *Using and applying mathematics* has to be embedded in the teaching and learning of mathematics.
- The skills of using and applying mathematics involve the organisation of thinking, the selection of ideas and strategies to try and the confidence to determine if these ideas and strategies will actually work (p.8)

## The five themes in the Using and Applying Mathematics strand

### 1. Solving problems (p.9)

Solving problems lies at the heart of mathematics. Children need to solve problems to become problem solvers. This may be stating the obvious, but it implies that children have to be given the time and space to tackle problems in mathematics lessons if they are to be confident and competent problem solvers. Problem solving should be integrated into mathematics teaching and learning, and become a regular part of the children's work. Devoting specific lessons to problem solving will help but embedding them into everyday lessons will provide the frequent and regular practice and consolidation children need. Problem solving should not be seen as a 'Friday only' activity.

In the renewed Primary Framework the problem solving theme focuses on problems that involve calculations set in wide ranging contexts as the children become more skilled and confident.

### 2. Representing (p.9 / 10)

Their pictures, diagrams, lists, tables or calculations become their working tools. Children use these tools, drawing on their mathematical knowledge, skills and understanding, to do the necessary mathematics and arrive at a solution.

Representation is a tool to aid thinking and the choice of representation is important in highlighting what does or does not meet the conditions or criteria that define the puzzle.

### 3. Enquiring (p.10)

Learners are engaged by successful and exciting learning. They become involved in finding out for themselves, asking and answering questions of their own and sharing what they have discovered with

others. The posing of questions is an element of creativity. It starts the process needed for the generation of ideas that are new to the learner, which takes them on a route of discovery.

This theme in the *Using and applying mathematics* strand involves children in following a line of enquiry. Initially the questions the children pursue may be given to them. As the children become more skilled at planning and organising their strategies and thoughts, and more confident at sustaining such activity, new questions will arise. These can be gathered, discussed and refined so that the children can pursue those that appeal to them.

The lines of enquiry may be drawn from any of the other six strands and, depending on the questions posed, involve varying degrees of decision making and reasoning. What is important is that children make informed decisions that they can justify, and begin to sustain a line of reasoning in support of their steps in the enquiry. The children need to be taught how to use pictures, lists, tables, graphs and diagrams to help them to organise and interpret any information they collect.

#### **4. Reasoning** (p.10 / 11)

Skills of recording... help children to see what *is* and begin to consider what *might be*. They help children to clarify what is the same and what is different. They help children to collect evidence when testing general statements or to predict and propose new hypotheses.

Making and using pictures, diagrams and notes to aid thinking are important skills for children, making decisions and deductions form another set of skills that children need to learn.

Using mathematical vocabulary and language of explanation and reasoning (helps children) record their explanations and reasons and refine these, using diagrams, graphs and mathematical notation and symbols.

Children need to see working examples and models that they can use, supported with opportunities to share and evaluate one another's approaches. Together, these provide children with the tools from which to make informed choices about how best to organise their work.

#### **5. Communicating** (p.12)

We record our thoughts to help us with the process and to keep track of the direction we are taking. The records we keep may be ad hoc or just annotations on a diagram or graph. They may be quite personal or idiosyncratic.

What we do record should support our thinking; in essence the communication is personal and relates to the doing of the mathematics. It is part of the process and may not be intended for a wider audience.

Encouraging children to interpret their personal communication (helps them) to see how effective it was in relaying their thoughts and ideas.

While children may recognise and use the pictures, diagrams and symbols involved in the above explanations and reasoning, they may struggle to apply them when asked to frame an explanation or reasoning of their own. This process requires those additional skills and the understanding learned through discussion and negotiation with others.

Building into lessons the opportunity to discuss, build and negotiate explanations and reasoning with other children is important.

The ability to communicate mathematical thinking, explanations and reasoning to others is only learned through guided practice.

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## Key Stage 1 *(p.13)*

### MA2: Number

#### Problem Solving

- Approach problems involving number, and data presented in a variety of forms, in order to identify what they need to do
- Develop flexible approaches to problem solving and look for ways to overcome difficulties
- Make decisions about which operations and problem solving strategies to use
- Organise and check their work

#### Reasoning

- Explain their methods and reasoning when solving problems involving number and data

#### Communicating

- Communicate in spoken, pictorial and written form, at first using informal language and recording, then mathematical language and symbols
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### MA3: Shape, Space and Measures *(p.13)*

#### Problem Solving

- Try different approaches and find ways of overcoming difficulties when solving shape and space problems
- Select and use appropriate mathematical equipment when solving problems involving measures or measurement
- Select and use appropriate equipment and materials when solving shape and space problems

#### Reasoning

- Recognise simple spatial patterns and relationships and make predictions about them
  - Use mathematical communication and explanation skills
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## Handling Data (p.15)

### Problem Solving

- Select and use handling data skills when solving problems in other areas of the curriculum, in particular science
- Approach problems flexibly, including trying alternative approaches to overcome any difficulties
- Identify the data necessary to solve a given problem
- Select and use appropriate calculation skills to solve problems involving data
- Check results and ensure that solutions are reasonable in the context of the problem

### Reasoning

- Explain and justify their methods and reasoning

### Communicating

- Decide how best to organise and present findings

## Official Reports

### **HMI: *Teaching of Written Calculations in Primary School. A Report by HMI (461) April 2002***

- **Calculations:** emphasised the importance of building links between children's mental and written methods

### **Ofsted: Summary Findings: mathematics, 2002**

- Pupils are less successful at using words and symbols to express a general rule and to derive a formula. Pupils of all ages have too few opportunities **to record mathematics** in a variety of forms and in a range of contexts, including problem solving. In number pupils are not always supported well enough in the use of informal methods of calculation when they cannot cope successfully with standard written methods (Ofsted Invitation Conference, Brunel University. March 20<sup>5</sup><sup>th</sup> 2002.)

### **Key NNS priorities for 2003/2004**

- The report identified a need to teach children **methods of calculation** that build on their mental strategies

### **QCA. Mathematics 2003/2004. Annual Report**

- Training is needed in pupils' recording of **calculations**. A number of terms are used to describe pupils' recordings, including 'jottings' and 'informal written methods'. Such terms are developing new meanings as numeracy consultants and teachers work with the ideas. Care should be taken in

publications at both local and national level to ensure that where terms are used they are exemplified (published March 2004)

### **Ofsted: Subject Report - Primary Mathematics (for 2003 – 2005)**

<http://live.ofsted.gov.uk/publications/annualreport0405/4.1.7.html>

- Data from this inspection cycle shows significant improvements in pupils' mental calculation skills, with the majority of pupils now able to draw well on a range of mental calculation strategies. In the teaching of written calculations, the position, though strengthened since 1999, remains that pupils' ability to use written methods of calculation for addition and subtraction is more secure than for multiplication and division. Division skills remain the weakest of the four operations

### **Ofsted: The Annual Report of Her Majesty's Chief Inspector of Schools 2005/06**

- The best teaching gave a strong sense of the coherence of mathematical ideas; it focused on understanding mathematical concepts and developed critical thinking and reasoning. Careful questioning identified misconceptions and helped to resolve them, and positive use was made of incorrect answers to develop understanding and to encourage students to contribute. Students were challenged to think for themselves, encouraged to discuss problems and required to work collaboratively. Effective use was made of ICT. In contrast, teaching which presented mathematics as a collection of arbitrary rules and provided a narrow range of learning activities did not motivate students and limited their achievement. For example, students could answer examination questions but could not apply their knowledge independently in different contexts (p.60)

### **Ofsted Report on the Foundation Stage (March 2007)**

#### **Differences in standards and achievement across the early learning goals:**

In mathematics, the key area of '**lower standards and achievement**' identified was **calculations** (p.7)'.

The inspectors recommend: 'children's low achievement in early reading, writing and **calculation** in some settings must be tackled so they are able to achieve the best possible outcomes to set them on the right path for their future education'.

## **Recent reports** (with reference to mathematics & numeracy)

### **Drummond & Moyles, 2004**

- A recent study of children's experiences in Reception classes highlighted a need for teachers to 're-examine the values and priorities of their approaches to numeracy teaching (p.27) calling into question the limited value of certain aspects of teaching and emphasising the 'low level of cognitive demand' (p.85)
- 'We recommend that reception class professional re-examine the values and priorities of their approaches to numeracy learning. With appropriate support from other professional and with a wealth of publications (for example, Worthington & Carruthers, 2003) practitioners should be encouraged to consider whether their current emphasis on the smallest and most basic units of numeracy learning is altogether appropriate' (p.27)

### EPPE - 2004

- A major new study of mathematics in the Early Years (EPPE: see TES October 29<sup>th</sup>. 2004) raises many questions concerning the way in which our youngest children experience mathematics and the way in which it is taught. It points to an emphasis on 'teaching to the test' and teachers' difficulties that result in lessons that 'fail to challenge and extend the thinking of pupils'
- Low-attaining boys are particularly likely to be disaffected if their lack of mathematical skills is exposed at an early age in whole-class question and answer sessions (see TES October 29<sup>th</sup>)

### EPPE – Review conducted by the Mathematics Education Review Group, Institute of Education, University of London

- 'The intention that whole class teaching needs to be 'interactive' and promote higher quality dialogue, discussion and strategic thinking, has not been realised. Indeed, there is some evidence to indicate that the increased use of 'traditional' whole class teaching with 'pace' is in fact undermining the development of a more reflective and strategic approach to thinking about mathematics, and may be creating problems for lower attaining pupils. (p.40)
- The overall enhanced gains in pupil competence may in large measure be a reflection of a closer match between what is being taught and what is being tested, rather than greater pupil gains in their understanding of mathematics.' (from the report, P.40)

### Anita Straker & Tim Coulson. TES.

- 'The job of raising attainment in maths in primary schools has been very well done, but is only half done... Progress is also harder because many of the issues that the numeracy strategy set out to tackle remain unresolved... Primary children still have **difficulties with calculation methods**, particularly compact written methods, and applying their understanding to problems, in particular those that require understanding of multiplication and division.' (December 17. 2004.)
-