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Solving problems together: emerging understanding

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This article describes the work of some children in a reception/year 1 class. A spontaneous discussion about a child's new shoes led to much interest among other children about their comparative shoe sizes and a suggestion from two children that we ask their families. This provided a genuine learning opportunity in response to their natural interest.

"What size shoes do you wear? Please sign your name below! Mums, dads, babies, childminders & grandparents welcomed "

Names, shoe sizes and comments recorded soon overflowed to two large sheets and following children's suggestions, subsequently involved other adults and children in school. Involvement in the survey provided interest and triggered a range of mathematical enquiries for several weeks. For the children whose idea it had been, it gave them - status when each morning they elected to stand by the classroom door, asking parents and siblings if they had recorded their shoe size. They were highly proficient in this, even writing a letter and photocopying it to give to children who were brought to school by a child-minder or one parent, whose other family members might have missed out

Some of the findings of our survey were analysed at “circle time” or in small groups: children commented on the frequency of names for some shoe sizes, their amazement that some adults wore the same shoe sizes (number) as some children and the relationship between shoe size and height, age and sex. Their survey also raised the anomaly of our sizing system for young children's shoe sizes and older children's and adults'. Interest was further supported by role play of shoe shops when a father brought in some shoe boxes for various sized shoes: this generated some worthwhile opportunities for mathematics in meaningful contexts, including measuring feet with rulers and tape-measures and using (real) money.

One child wrote to a shoe shop to ask what the biggest shoes in the world or, failing that, in the shoe shop were. Another child wrote to a clown at the circus that was showing in the nearby town, to ask the length of his shoes. A parent brought in a Guinness Book of Records: this encouraged two children to make their own books, recording the comparisons of children's measurements and of things in the classroom.

Matching maths to learners

We cannot predict what children will learn of what we directly teach. Allowing children to explore and practice skills in ways they have determined allows them to enter at their own level: significantly such activities also meet the requirements for tasks to be differentiated for learners' needs. I have found that the ingenious ways of working and individual means of recording children choose are well matched to what they understand. Teachers' suggestions derived from a more didactic mathematics teaching model may be less successful at matching learning to learners' needs, raising important questions about our expectations of children when they begin school.

The struggle to understand

Jamie and John decided to count the children's feet in the class. They carefully drew fifteen of the children, naming them aloud as they did so. Jamie checked how many they had drawn: the boys shared responsibility for writing a number by each figure beginning with 1 and counting correctly to 15.

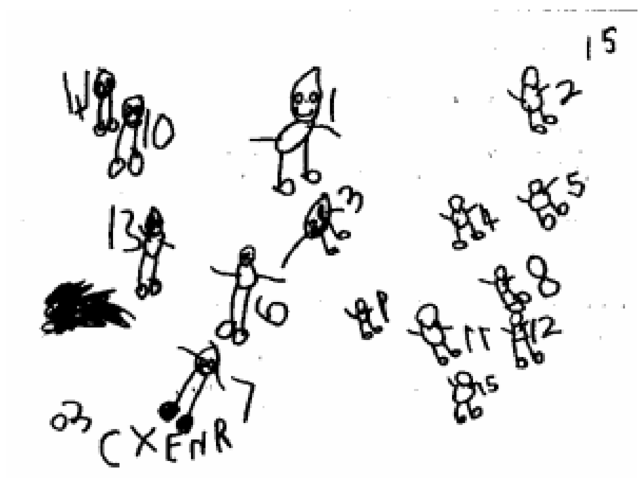

Figure 1. *Jamie & John found their own way to count the feet they had drawn.*

They recounted the children (fifteen) but could see that each child had two shoes: a difference between their findings and their expectations appeared to confuse them, suggesting to me that they recognised a higher number than fifteen was needed. It appeared also that they were unable to reconcile the “twoness” (of the shoes) with the “oneness” of each child. When they explained that they had difficulty finding out how many shoes there were, I asked if there was anything they could use to help them. After recounting and finding that there were still fifteen children Jamie collected some small wooden bricks. They placed one brick on each foot they had drawn: when they had covered all the feet in the drawing they removed the bricks and counted them.

Jamie and John had struggled with the concepts they explored and continued in their enquiry without any external pressure. Mental struggle is associated with new knowledge, although young children will opt out of learning activities if the activity is either too simple or too difficult. Whereas an adult could have told them to count the feet as they had the children, this was not obvious to them. I believe that it is likely that we will block effective learning if we are too ready to instruct children how to carry out a task from our adult perspective. Their choice and use of bricks permitted them to make concrete connections in ways that were accessible to them: it allowed them to bridge the discontinuity between what they already understood and their goal of greater understanding.

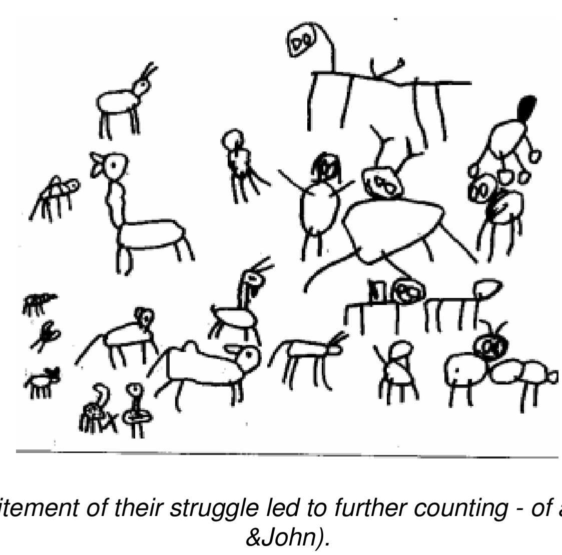

Figure 2. *The excitement of their struggle led to further counting - of animals' feet (Jamie &John).*

The following day whilst playing with farm animals, the two boys used a piece of green paper that had been a field for their farm play, and drew the animals. Jamie noticed that when drawing the chicken he had drawn four legs, and corrected it to two. This time they used the bricks and without hesitation counted the animals' feet as they had done the previous day, their "playful" approach on the second day indicating that they understood the concepts involved.

Jamie and John achieved a great deal in their task, moving forward from their current knowledge by using strategies that were entirely appropriate to their understanding. In the following example, Rose and Steven also struggled to build on their partial knowledge, and shared insights which helped them explore numbers.

"Numbers never stop"

Rose and Steven were interested in the numbers that had been recorded on our shoe survey. Steven, perhaps recalling a recent discussion on negative numbers when using the fridge thermometer, said he would count "forwards and backwards", explaining that "numbers could go on for ever" and that there were just "too many things to count in the world". Whilst the concepts of infinity and negative numbers had been explored (though not exhausted) previously, their idea of working backwards from zero was abandoned and they decided to see how far they could count forwards.

The children chose some white strips of paper and each child began writing on a separate piece of paper, beginning at 1 and stopping only when their last recorded number reached the end of the paper. The total number each child recorded differed, although at first they did not notice this. Steven's enthusiasm for reaching an undefined large-number goal also resulted in the size of his written numbers and therefore the totals on each of his strips also varying considerably. By the time I joined them the children had eight strips of paper covered with numbers from 1 to twenty or more - some over thirty. I asked how they would find their total - their original goal. The children were puzzled when they put strips in a continuous line and saw that joining them together - their first suggestion - would not result in a continuous line of numbers.

They found it hard to estimate how many numbers they had written in total, but adapted what they had been doing, deciding to write only numbers from 1 - 20 on each strip and discarding their earlier number strips. When they had a total of five strips of paper with numbers one to twenty on each, Steven represented them on a separate piece of paper.

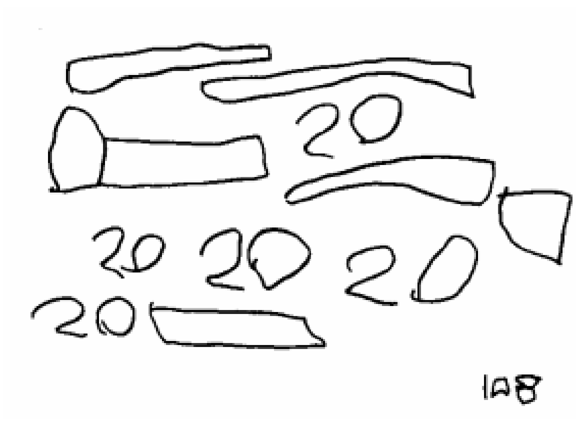

Figure 3. *Steven's response to their five strips of paper with numbers 1 - 20 on each: "There's about 108 altogether"*

He estimated that there was a total of 108 on the five strips but both children were uncertain how they could confirm the total. Rose's response shows how she was able to link her previous knowledge of counting in tens, to the current task.

Steven and Rose had already spent a long session on each of two days to explore their ideas of counting; interacting and negotiating their shared meaning. On the following day Steven settled down to make two further strips after which he confidently wrote one hundred and forty (10040): he had built on his previous knowledge through exploring ideas of counting in tens and twenties with Rose.

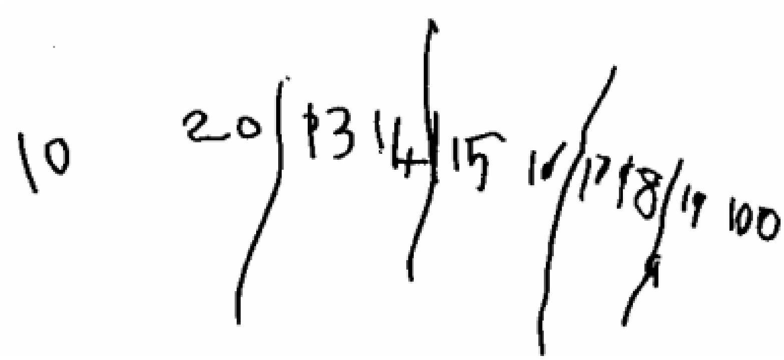

Figure 4. *Rose explained that the numbers between each line represented the twenty numbers on each strip of paper.*

The maths explored here included estimation and adaptation of known rules (counting in tens and twenties) and is impressive for such young children. Building on their partial understanding, they experimented with recording numbers and moved towards standard ways of recording: Rose's grouping of pairs of tens to represent counting in twenties, and Steven's subsequent recording of 140 as “10040” show considerable insight. Young children have a fascination for higher numbers and provided they can see a purpose, will grapple with highly complex ideas.

The examples given here certainly show the extent to which the children took ownership, modifying various aspects of their activity according to their needs, including:

- the focus of the task itself
- their ability to negotiate various aspects
- the order in which they would operate
- strategies used
- resources selected.

Just as the children working on ideas from the shoe survey responded in a personal ways to their enquiries, those in the following section selected ways of responding that made personal sense to them and which provided me with insights into their current understanding. Arising from one child's question, the following example also provided opportunities for others to use their own ideas and own ways of recording, and was very much `long on learning'.

"I bet there's a million seats in the train"

Following a school visit by rail to a market town, Lee remarked on the crowded carriage we'd travelled in. Discussing how we might find out the number of seats, one child suggested we could `phone the train people'. Once I had helped him dial the correct number for the local station, Lee repeated his question and returned to the classroom with an air of importance. He proudly informed the children that there were 75 seats in one carriage, adding "I bet there's a million seats in the train!" My suggestion that he might like to try and work this out was met with a clamour of offers from other children.

A number of interested children entered and explored this task, selecting ways of recording that were appropriate to them

In responding to Lee's question, it was clear that some children needed just to explore the idea of many seats in one carriage. Of the ten children's responses, one drew a person, others random shapes, a number 5 and some squiggles; the latter response is described by Hughes (1986) as idiosyncratic.

Five of the children, including Lee, responded with pictures, such “pictographs ... representing the appearance of (something) as well as its numerosity” (Hughes, 1986 p.57).

Figure 5. *Lee's pictographic response - 32 seats in the carriage: this was the maximum number he could squeeze into his drawing of a carriage. He said, "It's full."*

Most of these children drew seats in a carriage or people on seats: one went on to experiment with counting with tallies and then dots to represent seats, (moving from a pictographic to an “iconic response”).

The remaining four children used only iconic responses, based on one-to-one correspondence: they used circles or squares, sometimes checking their count by making a second mark inside the first. Helen used a range of responses and was the only child to use conventional (symbolic) number symbols as part of her working out.

Helen's final exploration began with a drawing of a carriage with seventy-five squares to represent the same number of seats. Remarking that she was "puffed out", Helen wondered if she could photocopy her drawing, explaining that she'd need "six more". When the seven sheets of paper were laid out across the floor, the children were all excited to see a complete “train” with equal numbers of seats in each carriage.

Figure 6. *In her first attempt, Helen wrote `75 seats' by the carriage at the top of the page: in the carriage below, she has drawn only seven seats.*

Figure 7. *Next Helen wrote 75, seven times. In the lower half of the page she wrote: "There's 5 in the carriage. There's seven carriages." It appears she was confusing 35 and 75. But she was very clear about what she did next, putting five Unifix bricks in each of seven trays she counted them. She drew five people then added to her page: "35 altogether."*

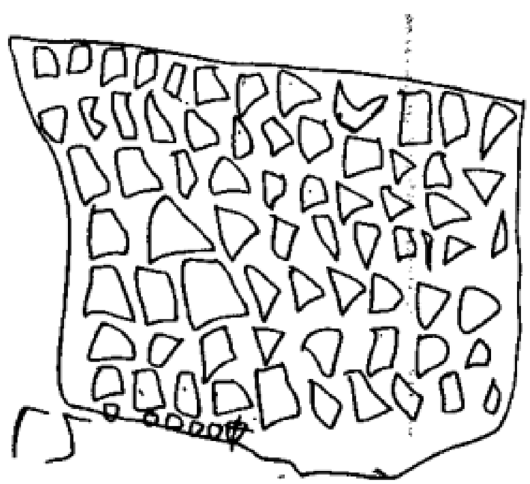

Figure 8. *Finally Helen returned to the first number she had used (75) and drew squares within a large square to represent seats in one carriage. She checked and finding that she had drawn 76 rather than 75, crossed one out.*

This had been a tremendous insight for Helen: it was also a very powerful representation for the children and sufficient for them at this point in their learning. Suggestions from several children to count all the seats, were difficult to achieve and none were at a point where they could cross over to standard forms of repeated addition or multiplication. However, within a genuine context, Lee's question had provided many possible ways of exploring challenging mathematical concepts through talk, concrete and visual experience and encouraged the children to take risks.

In terms of the National Curriculum, these examples fulfil the criteria for "using and applying" mathematics and demonstrate children's ability to be highly adaptive in situations in which they contribute to each other's learning. Well understood, an emergent mathematics approach fully meets the proposals of the Non-Statutory Guidance and offers opportunities for challenge and high levels of engagement in mathematics. The role of teacher is crucial, requiring demonstration, observation with understanding, sensitive support and open questioning.

Reference

1. M Hughes: Children & Number, Basil Blackwell, 1986