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Research Uncovers Children's Creative Mathematical Thinking

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Young children are amazingly talented, learning at rates they will never later exceed - in every area of their development. Understanding develops within the social and cultural contexts of their family and community. Their capacity for learning enables them to master the complexities of their mother tongue (and sometimes one or more additional languages within their family) in the first few years of life. They can reason, explain, describe, relate and argue and have a rapidly increasing vocabulary to enable them to explore and develop these skills. This ability to understand and use language makes it possible for them to also use marks to stand for something else - to develop graphical and symbolic languages such as drawing, writing and written mathematics.

Research exploring the development of children's early drawing and early writing has opened our eyes to what had been hidden. Understanding this development has helped teachers to recognise and value these early marks, to start from children's personal, informal understanding. When this happens, children respond in often highly creative and challenging ways and frequently exceed expectations. For children's understanding of the written language of mathematics, there is significant value in supporting and building on what they already understand.

Children's difficulties

Changes in mathematics and numeracy teaching in recent years has led to increased confidence in teaching and some gains for children. However for children, understanding of the abstract written language of mathematics continues to cause confusion for many. In is clear that there has been concern about children's difficulties with mathematics for a considerable time. In his significant study Children and Number: Difficulties in Learning Mathematics, Hughes identified the problems children experience in mathematics (1986). He showed how abstract symbols such as '+' and '=' and the abstract language (oral and written) of calculations such as '2 + 3 = 5' made no sense to young children. In his research older children continued to be confused since they had not build firm foundations: the 'school' mathematics was quite unconnected to their informal 'home' mathematics. It is this gulf between children's informal understanding and the formal written mathematics that has been so difficult to bridge.

Mathematical meaning

The problem with early 'written' mathematics is that a great deal of it has not been for real purposes, for real people (for an 'audience') or within a context that children understand. In a true sense it does not mean anything. Written mathematics has largely been produced by adults. The problem with worksheets is that they do not work. Their use has been extensive but has failed to help children better understand abstract written mathematics. This is a very real predicament that will remain unresolved until we are able to help children to bridge the gap. Children's ability to link early marks with meaning and to communicate their thinking through these marks is an important stage - both in becoming writers and mathematicians. Since the publication of Hughes's book, many studies have shown that young children entering school have a great deal of informal mathematical understanding that is largely unrecognised. The key question we endeavoured to answer was whether there was anything we might do deliberately to strengthen the links between children's informal (personal) and formal (abstract) mathematics. Our research led us to an extensive study of children's mathematical graphics.

What are children's mathematical graphics?

We coined this term to describe the full range of marks children make when exploring their mathematical thinking. They fall within the context of what is generally known as 'mark-making' in the Early Years (usually early writing and drawing). We analysed almost 700 examples of children's mathematical graphics from

children of 3 - 8 years whom we taught. The samples included very early marks (scribbles); personal or 'invented' numerals; drawings; icons such as dots, drawings or tallies; numerals and writing. We have been astonished by children's capacity for understanding, invention and creativity. It is time to re-evaluate young children's considerable abilities to understand the written language of mathematics.

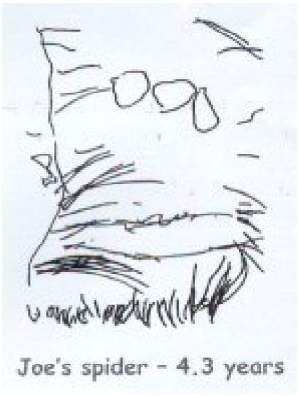
How can using their own ideas help children understand abstract symbols? Bi-numeracy

Using their own marks and making sense of what they do help children make personal meanings. When children's own mathematical graphics are shared and discussed, this enables children to become 'Bi-numerate'. This allows children to 'translate' between their informal 'home' mathematics and the abstract mathematics of school - to bridge the gap. Being Bi-numerate means that children can exploit their own intuitive marks and come to use and understand standard symbols in appropriate and meaningful ways. Developing personal written methods for calculations is an integral part of this (Worthington & Carruthers. 2003).

Analysing children's development

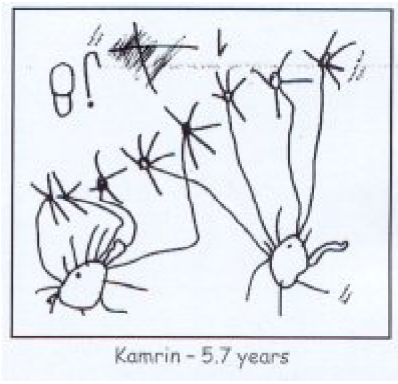
We analysed the forms (types) of graphics the children chose to use. Hughes's research (1986) had focused on some aspects of young children's early mathematical understanding, represented through their early, informal marks: we developed additional categories of forms based on analysis of our extensive sample. We then traced the development of mathematical graphics - essentially children's growing understanding of the abstract symbols and written calculations - the mathematics itself. Children respond in highly personal ways. The categories or 'dimensions' we developed highlight the children's development which arise within children's earliest explorations with marks and lead to explorations of written numerals in highly individual ways. Young children represent numerals; they represent quantities (of things) that they have not counted and other things that they do count.

Joe's spider


Joe's spider - 4.3 years

In the nursery Joe (age 4yrs 3months) was playing with soft toy spiders. He later chose to draw a picture of a spider (see figure 1). He showed his drawing to the teacher and said, 'My spider has got eight legs.' He had drawn the spider with many more than eight legs. Looking at the spider he saw lots of legs and represented that idea in his drawing. Joe showed a growing awareness of number and quantity and was able to describe it. He knew that a spider has eight legs and you can represent that idea in your drawing. Joe has used numbers in his own meaningful way. In our study we put this form of graphics in the dynamic range. There is liveliness in Joe's spider. He was uninhibited in his demonstration of his knowledge of spiders and number. We want to keep this mathematical thinking as Joe progresses through his education. The mathematics he used is what we have termed dimension 4a, 'representing quantities that are not counted'. This early development ensures firm foundations for early addition and subtraction, explored in ways that make sense to the child. One fascinating area that was revealed is the way in which children explore symbols in non-standard ways, by implying or inventing symbols that can be read as though the symbol was there. This understanding leads to calculations that also include multiplication and division.

Kamrin's Tweedle birds



Kamrin - 5.7 years

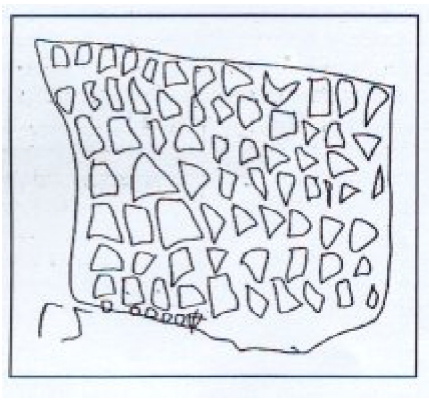
Kamrin (5years 7months) was in a reception class. The children had been introduced to several forms of division and the teacher had modelled the children's ideas of division. Kamrin chose the number eight to work out if it could be shared equally without leaving a remainder. Kamrin represented this in his own unique way (see figure 2) . He created the 'tweedle birds', giving each bird four eggs. He then wrote the numeral eight and a question mark. At first he decided that he could not share eight equally and put a cross. He checked this and then decided that he could do so. He scored through his cross and put a tick beside the cross. Kamrin used a combination of symbolic and iconic forms of graphics. He used early mathematical operations which we have termed the fifth dimension. Kamrin's use of his own mathematical graphics had moved on from Joe's since he not only progressed beyond representing quantities but used his knowledge of counting to find out about division. He has also skilfully self-corrected.

When calculating with larger numbers children may also decide to integrate the 'jottings' they have been taught (such as the use of an empty number line) and sometimes draw on earlier strategies.

Frances and the train

The reception / Y1 class had visited a market town by train. On our return, Aaron remarked: 'I bet there were a million seats on the train!' We talked about how we might find out and after some discussion one child suggested we phone 'the train people'. Aaron phoned and told us there were 75 seats in each carriage and 7 carriages on the train. I used this as a meaningful problem to solve, inviting children to find their own ways of working out how many seats there had been altogether on the train. Ideas and graphical responses were chosen by the children and were highly differentiated. Some chose to draw a single carriage filled with as many seats as they could fit in and others used iconic responses such as squares or dots to represent seats. Two children decided to explore repeated addition by selecting some large scallop shells and placing two wooden beads in each. Frances, 6:1 years, used a range of responses, exploring several different ways including '75' written seven times. She then drew a carriage with 75 seats and decided to re-count (check) what she had done. Finding that she had represented 76 seats she crossed one out.

I was impressed by Frances's ability to represent the 75 seats accurately. Although I did not expect her to multiply 75 x 7, I wondered if she saw any possible next steps towards a solution. Smiling, I remarked 'but there were seven carriages.' For a moment Frances looked puzzled, then burst out excitedly 'the photocopier!' and explained she'd need 'six more'. When the additional six copies were laid on the floor with her original, this was a very powerful representation of repeated addition for all the children. When Frances begins to explore more standard forms of multiplication she will have a deep level of understanding on which to build.



Frances and the train - '75 seats' (developed to '75 x 7')

Does creativity have a place in mathematics?

The highly individual ways in which children explore their mathematical thinking on paper helps them make

sense of what must appear to them the alien language of written mathematics. Creativity in mathematics education relates to the processes and particularly the personal and thinking skills that children can develop which will support deep levels of learning. It has been described as: 'the quality of the thinking taking place - a breakthrough in thinking - a creative moment and significant to learning' which can 'initiate powerful and challenging learning experiences and harness the imaginative power of individuals' (Talboys. 2003). Creativity extends to all aspects of the curriculum, to all aspects of human development - including mathematics. The QCA's guidance for the Foundation stage emphasises: 'Effective teaching requires practitioners how help children see themselves as mathematicians' (2000. p. 71). For children to become (young) mathematicians requires creative thinking, an element of risk-taking, imagination and invention - dispositions that are impossible to develop within the confines of a work-sheet or teacher-led written mathematics.

Thinking and creativity - meaning making

Creativity has often been linked to the arts such as painting, music and dance. We need to think wider than this - in every area of the curriculum children can be encouraged to think creatively - to innovate, to take risks and tackle problems in their own ways. The National Curriculum 'Handbook for Teachers' proposes that 'Creativity is a skill that needs to be promoted across the curriculum. Creative thinking should enable pupils to generate and extend ideas, to suggest hypotheses, to apply imagination and to look for alternative outcomes'. (QCA. 1999a. P.22)

When teachers encourage children to decide how they will 'put something down on paper' and listen to what children say about their marks, they are provided with new insights into children's development. Children need to make sense in their own ways rather than colouring-in ours. And young children are very good at making their own sense through their own marks and symbols, as these examples show. The dominance of worksheets - highlighted by OFSTED as a key issue in a recent report - will never change unless practitioners support children's own ways of exploring their mathematical thinking so that they make strong connections with their own understanding.

Official guidance

Valuing children's own ways of representing their mathematical understanding is supported by a growing body of research. The government are keen to promote more flexible ways of working, and innovation and creativity are encouraged. The Curriculum Guidance for the Foundation Stage (mathematics) emphasises:

- It is important that children's experiences of mathematics are enjoyable and meaningful and that their confidence is always fostered through building on what they know, understand and are able to do
- Effective teaching requires practitioners who maintain children's enthusiasm and confidence when they begin to record their mathematics. Asking children to 'put something on paper' about what they have done or have found out will allow them to choose how to record or whether to, for example, use a picture, some kind of tally or write a number. Children are most likely to want to 'put something down' when the record has some purpose for them'. It will provide a much stronger foundation for mathematical development than an introduction to 'sums' such as '2 + 3 =' before children have had opportunities to explore number situations, count, begin to calculate, talk about how they worked things out and record in their own ways. (QCA. 2000. p. 71/72). The National Numeracy Strategy makes a similar emphasis:
- At first, children's recording may not be easy for someone else to interpret, but they form an important stage in developing fluency (p.12)
- Parents should understand that children will need to start with unstructured personal jottings that should be valued. These jottings are an essential step towards conceptual understanding. As children gain understanding, their written methods become more fluent and more efficient (p.18.)
- Children will need to have plenty of experience of using their own individual ways of recording addition and subtraction activities before they begin to record more formally.
- Children should be encouraged to talk and write about their work in their own way.
- It is easy to be misled by children who start to use standard forms of recording too early, into thinking that they necessarily understand what they have written (p.19)
- A mixture of words and symbols will be used by children to explain by someone else the mental methods they have used
- Children will use a variety of ways of recording addition and subtraction, reflecting the variety of

mental methods used (p.20)

Different methods are often used for the same calculation and this can lead to useful discussion (p.21), (QCA. Teaching Written Calculations. The National Numeracy Strategy. 1999b). The recent Primary Strategy also emphasizes creativity in numeracy and from September 2003 one of the priority areas for consultants is ‘fostering creativity’ (Collins, 2003).

Creativity and mathematics: children learning, teachers teaching

- Being able to recognise marks that relate to children's mathematical thinking helps teachers determine ways in which they can further support children's mathematical graphics
- Recognising different forms of graphics and the mathematical dimensions (categories) enables teachers to assess children's mathematical development
- This understanding helps teachers determine practical ways to help support children further. In addition to helping children understand abstract written mathematics, we have identified other significant factors:
- Transitions: supporting children's mathematical graphics can aid transitions between home and nursery or school; nursery to school and between the Foundation stage and KS1 by ensuring that the mathematics they explore at each stage builds on what they already know and can do
- Inclusion: all children can explore their mathematical thinking and reach out to their highest levels of thinking through their mathematical graphics. This is as true for children with identified special needs as for the most gifted.

Conclusion

The extensive research we have carried out points to ways of working with young children that provide an alternative to worksheets and supports children's creative thinking in mathematics. Our findings show the extent to which encouraging more open ways of working are possible. Our evidence is that children's mathematical graphics helps children become Bi-numerate and supports significant learning.

Starting points

Understanding, supporting and developing mathematical graphics raises pedagogical issues. How can teachers tune into children's thinking in this way? Here are some useful starting points:

- Make sure your graphics area has mathematical resources
- Be aware of the mathematics. What appears to be writing and drawing may well be mathematics
- Provide mark-making equipment in play areas both inside and outside the classroom
- In focused small and large group time encourage individuals to decide how they will represent their thinking

When children respond in a variety of ways then you will know that you have begun to create a culture that encourages children's individual mathematical thinking.

Creative mathematics

QCA proposes that practitioners need to support children so that they are ‘able to communicate ideas and feelings, make connections, innovate and solve problems. It begins with curiosity and involves children in exploration and experimentation. As the express their creativity, they draw upon their imagination and originality. They make decisions, take risks and play with ideas. Children's creativity develops over time and takes time. It is best facilitated by adults who sensitively support this process and do not dominate it. If they are to be truly creative, children need the freedom to develop their own ideas and the support of adults who can help them gain the skills that enable their creativity to have expression’ (QCA. 2000. p.118).

Children's mathematical graphics provide opportunities to explore and develop these skills. It is time that we all spoke out as advocates for creativity within children's early 'written' mathematics.

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