Maulfry Worthington and Elizabeth Carruthers

Emergent Mathematics: a Collaborative Approach

Mathematics Teaching. 162. March 1998. pp. 11 – 15.

The dynamic nature of an emergent approach to learning offers children and teachers considerable advantages, extending opportunities for young children beyond narrowly defined adult criteria. Whitebread (1) has described this approach as "potentially a very exciting and powerful set of ideas".

Encouraging children to explore talk and record in their own ways permits exploration of mental processes and underlying themes, allowing connections to be made with previous understanding. Learners construct deep levels of understanding in a way that they can never do when answering adult-posed questions or work-sheets. Such an approach to learning is more likely to be readily accepted with reference to English or art: mathematics of all the subject areas has been the last to be explored in relation to learners and learning in this way.

Emergent or developmental writing

There is a prolific amount of literature supporting an emergent or developmental approach to children's reading and writing. Opposing previous behaviourist models, the emergent approach has been almost revolutionary in its influence on materials and methods. Hannan wrote this in the TES of June 28th 1996:

> "The classroom practice of tens of thousands of Key Stage One teachers has been changed by the findings of emergent literacy. Teachers are better equipped conceptually to exploit children's knowledge of reading and writing as a bridge to what is conceptually required".

Some of the earliest, and most influential of these literacy theorists included Goodman (2) and Smith (3). They have taken a psychological approach to the teaching of literacy skills. Their writings are based on observations of young children engaged in writing and reading behaviours. Previous practice had taken a subject approach, in which the English language was broken down into parts in order to make it simpler for children to understand, and the amount taught at one time was restricted. Smith (3) argued that this actually made it harder and stressed that it appeared nonsensical to children. Holdaway has referred to such practice as "criminal print starvation".

Emergent literacy has certain common attributes:

- A child exhibits behaviours that demonstrate she is acquiring conventional knowledge of reading and writing in a gradual way, (this is the stage that Holdaway named 'emergent').
- Children's approximations are accepted.
- The developmental process is viewed as a continuum from birth throughout life.
- Teaching is based on the observations of children's learning and behaviours: teaching and learning are therefore inter-twined.
- Children are seen as powerful learners, constantly making sense of their world.
- Learning is most effective when it is experienced as a whole picture which is not being broken into meaningless parts.
- Learning is best when it is presented in meaningful contexts.
- Children have real choices within their learning.
- The role of the teacher is not as sole giver of knowledge, but she understands that environmental and social factors and the child's own knowledge are important contributory factors to the learning process. She sensitively takes these factors into consideration.

Whole language background

In common with a number of other teachers, most of our group (the Emergent Mathematics Teachers) during the last decade were deeply involved in whole language approaches, including emergent writing. We had seen children's understanding develop in ways that we had not previously believed possible, their insights appearing to be at a far deeper level at a much earlier age. Some of us had collected and analysed thousands of examples of children's early writing, thrilled to discover that children shared similar developmental pathways as their understanding and knowledge grew. Knowledge of writing, spelling and punctuation appeared to be constructed by individual children, as they used language in social situations and for genuine, meaningful purposes. We had seen how our open questions helped. We had seen how children's understanding was enhanced by demonstrations, or modelling, of our writing for authentic, shared purposes. As our appreciation grew through daily practice, reflection and discussion, we sought out literature to further our understanding. We had found similar developmental pathways and subsequently read of published research which showed similar findings.

As experienced early childhood teachers we had believed that the practice that we provided was child-centred, in settings that provoked enquiry and valued play. However, most of us individually also felt some unease about the way in which we taught mathematics and the about the early experiences we provided for children. Mathematics, we believed, was less well grounded within the children's own experiences; activities were more contrived and less stimulating and challenging; understanding shallower. We were also aware that, whereas in English children were attaining levels beyond what we had previously expected, this was not so in mathematics. Of particular concern was our recognition that most children experienced difficulties in the transition from home to school, where mathematics was more abstract and formal; this is a problem explored in considerable depth by Hughes (4) and also Whitebread (1). MacNamara (5) and Aubrey (6) studied pre-school children's number knowledge and the teachers' expectations as revealed by the activities they offered. These expectations were found to be very low compared with what the children were actually capable of. Aubrey indicated that reception children may not be able to use the conventions for representing what they know, but they have already acquired much of the mathematical content. Rogers-Ewers & Cowan's study (7) supports this, showing that children's understanding of mathematics is more advanced than their ability to represent numbers in standard form.

Most of us used published maths schemes and a range of practical activities with classroom mathematics resources: we were conscientious and yet our teaching remained uninspired. Several of us were also less than confident about mathematics as a subject, and so published schemes acted as a prop for us, obviating the need to think a lot about either the mathematics or the next steps for the children.

One teacher who has been a member of the group since its inception, had already registered a proposal for a thesis on emergent mathematics, having gained sufficient insight to believe that such an approach might offer rich possibilities for understanding young children's construction of mathematics. As individual teachers we were all questioning our practice and interested in making improvements.

Against this background in 1990 Rex Stoessiger a mathematics researcher from Tasmania, came to Britain for a year, dividing his time between Cambridge where he was working with Hilary Shuard, and Exeter University. Rex knew that many aspects of emergent literacy teaching could be "flipped over" into mathematics (8). We explained his idea to classroom teachers who had a good of emergent literacy approaches, and who were interested to explore this possibility. A small group of teachers subsequently came together and since that time we have developed the Emergent Mathematics approach.

Others have explored similar approaches. Whitin, Mills and O'Keefe (9) researched a first grade class in the United States over a period of a year, using an emergent approach. The researchers claimed the there was a shift in emphasis from computation to understanding mathematics in meaningful contexts. Atkinson (10) provides classroom-based answers to the problem of supporting emergent mathematics through a collection of teachers' stories. Atkinson makes a very clear connection between emergent literacy and mathematics, drawing also on the research of Hughes (4) which forms the basis of her book. More recently, Whitebread (1) has also placed emphasis on the emergent approach, which he illustrates in a concise theoretical way with reference to psychological research. Stoessiger and Wilkinson (11) point out the parallel with what real mathematicians do, "engaging with major mathematical ideas through open-ended and challenging activities".

They also refute allegations that the teacher's role is laissez-faire: "…teachers are continually assessing

and providing suitably challenging activities, demonstrating standard forms and asking questions which may prompt the student to clarify, to predict, to develop further, to look for alternatives" (11).

Developing our approach

Rex's visit had been the catalyst for providing mathematics-rich environments for children: this included "publishing" children's mathematics – an idea that at one time was much more acceptable with children's writing. Rex also used teacher-written "challenges" that provided the stimulus for children's mathematical explorations. Although this was a helpful starting-point, through observation we moved towards encouraging the children to initiate their own mathematics, and challenges provided by adults were abandoned. Over a long period we cautiously experimented and evaluated, developing our emergent approach to mathematics that has been the focus of the group during the past seven years.

Through extensive discussion and study our understanding has grown. There is as yet only a small number of published articles and books available, including discussion papers and teachers' accounts of similar approaches. Emergent mathematics is currently in the position that emergent writing was twenty years ago.

Grounded Theory

The Emergent Mathematics Teachers' approach, because it is centred on teaching and on children and because it also draws on the developmental literacy approach, is a grounded theory. For the past seven years this teachers' group has been involved in hundreds of hours observing and participating with learners, engaged in using and learning mathematics in home, nursery, classroom and playground settings. The Emergent Mathematics Teachers' group has regularly discussed children's mathematical recordings and actions, and the important influence of their families. We have talked with parents, sharing ideas and understandings about mathematics. The children and their families are at the heart of the mathematics. Every year children and their families are at the centre of our conferences. Our theory is neither an ivory tower theory nor an armchair theory but is a living theory which is constantly updated and influenced by all around.

Social constructivism

The teachers who have developed this approach also claim a social construction basis. Sophian (12) suggests that there are three dominant theoretical perspectives.

- Piaget stresses the importance of invention in cognitive development. Children's own mathematical constructions are viewed as more important in the child's understanding than the theories projected by adults or others onto the child.
- Domain-specific theorists see cognitive development of many different domains of cognition, such as language and number and knowledge about causality. Within each domain, learning is grounded on an inborn framework of principles, which direct the child's attention to parts of their experience that are of particular importance for learning about that domain.
- The social-cultural perspective is based on the work of Vygotsky, who highlighted the importance of cultural practices on cognitive development. Proponents of this theory forward that cognitive development is a process of acculturation into the child's understandings of the world within society, cultural habits and conventions.

Social constructivism fits into all thee above perspectives. From a constructivist view, knowledge is a personal construct of the learner. Children make their own meanings, building up their own understandings. Knowledge is not passively received but actively built up by the learner: learners are not passive recipients of "the facts" but active developers of their own networks of concepts and theories. A social constructivist stand would strongly emphasise that knowledge is gained within social and cultural contexts. This challenges Piaget's model of the child as lone scientist, but supports the Vygotskian view that the child learns from "knowledgeable others".

Teachers socially constructing their own knowledge

The social aspect of learning is a strong one within the Emergent Mathematics Teachers' group, as we also believe we learn about children's learning, not only by observing children, but also by the strong social interactions through meeting and discussion within our group. It is this effective team approach that has supported our knowledge and is the backbone of our learning. It is the basis from which we exist as a group.

Increasingly teachers' voices have been diminished over recent years, because teachers have been pressurised to accept a prescribed curriculum. As a group, we hold firmly to the belief that we need to think for ourselves and because of this we find it easy to give this freedom to the children. It is difficult to have an emergent approach in an educational climate that disempowers teachers. Teachers as well as children need to feel self-worth and this needs also to extend to the world of educational research: as well as being subjects of research, teachers deserve the respect of being equal partners in the research relationship. We therefore do not want to have research done to us without acknowledgement. The power of collaborative research can be dynamic for both sides but it is difficult for creative teachers to survive on their own; together they are a potent force.

We may also be unusual in that we have had no assistance or funding from either universities or local authorities, and yet together we have managed to retain our enthusiasm and commitment through changes of posts and times of tremendous reorganisation and pressures within the educational system.

Our collaborations – teaching, innovation, reading, research, writing, INSET and conferences – have been a journey. The journey has been arduous and fun, challenging and tortuous. Whilst we have not worked in the same schools, we have shared certain aspects of our experience and an enthusiasm for education. We have also shared a concern to update and improve our practice in order to provide benefits to the children. We have developed a way of teaching mathematics that considers the child's perspective. As a result of our collaboration individual teachers' confidence and interest in mathematics has soared.

Integrating innovations

As we clarified our ideas about an emergent approach in mathematics, we also generated ideas about practical issues. We share a common philosophy about the way we teach, although we have never expected to produce a list of requirements on "how to teach using an emergent approach".

The point already made about our own social construction as teachers is highly significant: we can learn both from colleagues and from teachers beyond our locality. This innovative framework has always been sufficiently flexible to allow us to embrace other initiatives and learn form other research. For example, we have all been sufficiently interested in providing children with daily opportunities to initiate their own activities in the nursery and at key stages one and two, and our classrooms have provided self-service environments that supported this. Therefore, in mathematics as in all aspects of an integrated early childhood curriculum, children are able to initiate their own mathematical activities and select materials they deem appropriate. Equally, there will be times when there will be small-group activities when the teacher interacts directly with the children to support concepts and skills. When the National Curriculum was introduced we found that "Using and applying mathematics" fitted naturally with our existing practice. The Non-Statutory Guidelines also specifically recommend that children "develop their own methods" and that flexible approaches including "standard and non-standard written symbols" are used.

What became gradually apparent was that our thinking about mathematics learning was also being influenced by other aspects of education that were of interest to us. Developing communities of enquiry through an involvement in "philosophy for children" led naturally to some deep discussions on the nature and purpose of number and on infinity. An interest in philosophy and thinking skills (14) also assisted us in more open questioning and a willingness to see that mathematics could be approached from different perspectives by different learners. The methods children used and their ways of reasoning assumed greater importance than a quick "right" answer, though we have never accepted casual responses. We encouraged children to search for rules to justify what they were doing and to look for patterns and discrepancies. One of the most significant changes in our role has been a shift in the way we handle "errors": these are viewed as providing information about the learner's current understanding and points for discussion. This may be compared to Goodman's view of reading miscues (2), assisting children to find patterns and make connections, and also providing feedback for teachers. The search is for meaning and deep levels of thinking.

Probably the greatest influence on what we have been doing has been our study of children's mathe-matical schemas, through the work of Chris Athey (13). This challenging study focused on nursery children's actions, talk and representations as they explored personal theories or schemas that are “systems of thought and perception” (13, p. 49). Significantly, many of the schemas that children explore are the basis of later concepts and include many mathematical ones. Through direct classroom observation we were able to gain further insights into ways in which we might support children's deep mathematical interests. Involving parents in noticing their children's schemas at home enriched our understanding, raising parents' awareness of mathe-matical potential in hitherto unsuspected activities. Perhaps this is one of the crucial aspects of this way of working, since observing a child explore her deep personal interests in mathematics allows us to peep through a window into the her mind whilst the learning is constructed. Children's partial under-standing and alternative ways of seeing can provide teachers with information about the contribution they can make, as children move with confidence towards greater understanding.

This approach to mathematics has been referred to as “whole mathematics”, “mathematical literacy”; “natural learning and mathematics” and “mathe-matics with reason”. We have debated many times about the most appropriate title. Eventually it was agreed that the most acceptable is `emergent' mathe-matics, since this term is widely accepted and understood with reference to writing. Significantly this term is also becoming an accepted form in the literature (eg 10, 1). The term `positive' also under-scores everything we believe: the following principles outline the philosophy.

A positive approach to mathematics:

- This approach values children as learners and also values the prior understanding which they bring to the nursery, playgroup or school setting.
- It values the part which children's families & home environments play in supporting and promoting mathematical understanding.
- Learning is viewed as a partnership between the child, home and nursery, playgroup or school.
- It is a human approach to learning, recognising that children build their own understanding through interactions with others.
- It requires a responsive approach to children's active and enquiring minds.
- It is respectful of learners and says "you can" to each child. Importantly such an approach also says "you can" to teachers.
- It puts the child at the centre, with teachers helping children uncover the mathematics rather than merely covering the curriculum.
- A positive approach recognises the value of mathematics for real contexts and for real people. Children negotiate meaning and are involved in genuine discussions. Learners are involved in setting tasks, selecting resources and deciding appropriate ways of representing their understanding.
- This approach recognises the significance of children's schemas in developing mathematical concepts and the importance of play in providing for diversity of needs, strengths and construction of understanding.
- A cycle of child's play and self-initiated activities, adult observation and assessment inform plans for focused, adult-initiated activities.These in turn further support and extend mathematical understanding.
- This approach requires rich learning environ-ments - physically, psychologically and philosophically. Such an approach does not avoid current issues in education but also focuses on the needs of learners.

This is provocative maths, that is to say it inspires, motivates and challenges children's minds. It requires them gradually to make existing percep-tions explicit, to try out alternative ways of thinking, looking and representing. Posed at our first confer-ence, the following question has remained uppermost in our minds:

> "What do you believe you must do deliberately, if your teaching is to support children's mathe-matical thinking?"

Reflecting on the complex and often difficult process of developing this approach, we recognise that we still have a long way to go, and welcome others' perspectives.

References

1. D. Whitebread, 'Emergent Mathematics or how to help young children become confident mathematicians' in J. Anghileri (Ed.): *Children's Mathematical Thinking in the Primary Years,* Cassell, 1995

2. K. Goodman: *What's Whole in Whole Language?* Scholastic, 1986

3. F. Smith: *Reading,* Cambridge University Press, 1978

4. M. Hughes: *Children and Number,* Basil Blackwell, 1986

5. A. MacNamara: `How many are there?' *Early Years*, Vol. 13. No. 1, 1992.

6. C. Aubrey: An investigation of the mathematical knowledge and competencies which young children bring into school, *British Education Research Association,* Vol. 19. No. 1, 1993

7. J. Rogers-Ewers & R Cowan: 'Children as apprentices to number', *Early Child Development and Care,* Vol. 125. pp 15 - 25, 1996

8. R. Stoessiger: 'Using language learning conditions in mathematics'*: a paper presented at the Primary English Teaching Association,* Royal New South Wales, Australia, 1988

9. D. Whitin, H Mills &T O'Keefe: *Living and Learning Mathematics,* Heinemann, 1990

10. S. Atkinson (Ed): *Mathematics with reason,* Hodder & Stoughton, *1992*

11. R. Stoessiger & M Willanson: 'Emergent mathematics', *Education 3 - 13 Years,* Vol. 19, pp 32. 36, 1991

12. C. Sophian: *Children's Numbers,* Brown & Benchmark, 1995

13. C. Athey: *Extending Thought in Young Children: a parent-teacher partnership, London*: Paul Chapman Publishing, *1990*

14. M. J. Coles & W. D. Robinson (Eds): *Teaching Thinking,* Bristol Classical Press, 1989